The Boundary Layer and Related Phenomena

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Overview

The Boundary Layer and Related Phenomena
    Reynolds Fluxes and Their Physical Intepretation

Turbulent flux of momentum and turbulent stress

The variances and covariances ($\overline{u'u'}$, $\overline{u'v'}$, $\overline{u'w'}$, $\overline{v'u'}$, $\overline{v'v'}$, $\overline{v'w'}$, $\overline{w'u'}$, $\overline{w'v'}$, $\overline{w'w'}$) that appear in the Reynolds-averaged Navier-Stokes equations are the components of the *turbulent kinematic momentum flux*.

Turbulent flux of momentum and turbulent stress

This name becomes apparent if we consider products of these quantities and density, *e.g.*,

$$\rho\overline{u'w'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{kg\ ms^{-1}}}{\mathrm{m^2\ s}}\right] = \left[\frac{\mathrm{kg\ m}}{\mathrm{m^2\ s^2}}\right] = \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right].$$

Turbulent flux of momentum and turbulent stress

$$\rho \overline{u' w'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{kg\ ms}^{-1}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{kg\ m}}{\mathrm{m}^2\ \mathrm{s}^2}\right] = \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right].$$

The above term, for instance, has a meaning of x component of momentum transported in average by fluctuating velocity component w' per unit time per unit area of the surface normal to z axis.

Turbulent flux of momentum and turbulent stress

$$\rho\overline{w^{'}u^{'}} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{kg\ ms}^{-1}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{kg\ m}}{\mathrm{m}^2\ \mathrm{s}^2}\right] = \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right].$$

On the other hand, this term may also be interpreted as z component of momentum transported in average by fluctuating velocity component $u^{'}$ per unit time per unit area of the surface normal to x axis.

Turbulent flux of momentum and turbulent stress

$$\rho\overline{w'u'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{kg\ ms}^{-1}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{kg\ m}}{\mathrm{m}^2\ \mathrm{s}^2}\right] = \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right].$$

Thus, these fluxes can be thought of as the transport of mass per unit area per unit time. In other words, they represent the force per unit area.

Turbulent flux of momentum and turbulent stress

Physical quantities having opposite signs to the momentum flux components are components of the turbulent stress:

- $\tau_{xx} = -\rho\overline{u^{'}u^{'}}$
- $\tau_{xy} = -\rho\overline{u^{'}v^{'}}$
- $\tau_{xz} = -\rho\overline{u^{'}w^{'}}$
- $\tau_{yx} = -\rho\overline{v^{'}u^{'}}$
- $\tau_{yy} = -\rho\overline{v^{'}v^{'}}$
- $\tau_{yz} = -\rho\overline{v^{'}w^{'}}$
- $\tau_{zx} = -\rho\overline{w^{'}u^{'}}$
- $\tau_{zy} = -\rho\overline{w^{'}v^{'}}$
- $\tau_{zz} = -\rho\overline{w^{'}w^{'}}$,

Turbulent flux of momentum and turbulent stress

Note, $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$.

Here, τ_{xx}, τ_{yy}, τ_{yy} are the normal components of the turbulent stress, while $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$ are the shear components of the stress.

Why is this called a stress? Why do we describe the components as normal and shear?

Turbulent flux of momentum and turbulent stress

Think about an idealized cubic volume.

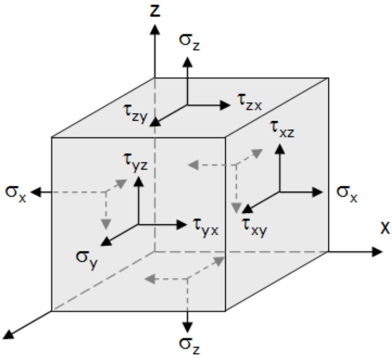
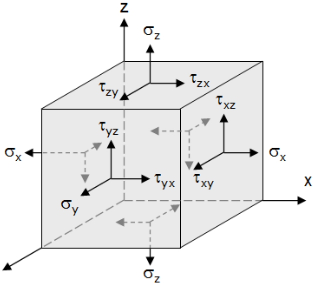


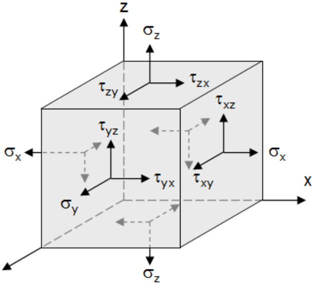
Figure: Idealized cubic volume indicating turbulent stresses. Note here, *e.g.*, $\sigma_x = \tau_{xx}$.

Turbulent flux of momentum and turbulent stress



$\tau_{xz} \rightarrow$ the flux of the momentum parallel to a volume face (*e.g.*, of u' through a boundary of a cubed volume by w') causes the parallel momentum (u' in this case) to change at that face. Accordingly, the momentum flux is the *stress* applied at this face.

Turbulent flux of momentum and turbulent stress



We therefore call the momentum fluxes in the momentum equations the *Reynolds stresses*.

Turbulent flux of momentum and turbulent stress

Note, that in meteorological literature, the turbulent momentum flux and turbulent shear stress are usually normalized by density (*i.e.*, $-\overline{w'u'} = \tau_{zx}/\rho$).
These normalized quantities are often called *turbulent momentum flux* and *turbulent shear stress* with the word *kinematic* being omitted.

Turbulent flux of momentum and turbulent stress

We now apply the notion of turbulent stresses and momentum flux in the Reynolds-averaged momentum balance equations

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} + \overline{w}\frac{\partial \overline{u}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x} - f\overline{v} = 0$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + \overline{w}\frac{\partial \overline{v}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial y} + f\overline{u} = 0$$

$$\frac{\partial \overline{w}}{\partial t} + \overline{u}\frac{\partial \overline{w}}{\partial x} + \overline{v}\frac{\partial \overline{w}}{\partial y} + \overline{w}\frac{\partial \overline{w}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial z} + g - \beta\overline{\theta'} = 0$$

Turbulent flux of momentum and turbulent stress

In models of atmospheric flows (including boundary-layer flows), which are essentially turbulent (with the exception of some special flow cases), viscous terms on the right-hand sides of our original equations are usually neglected.

This is a reasonably safe assumption in turbulent boundary layer flows since the effects of molecular diffusion are much smaller than the effects of turbulent eddies.

Turbulent fluxes of heat

The covariances ($\overline{u'\theta'}$, $\overline{v'\theta'}$, and $\overline{w'\theta'}$) that appear in the Reynolds-averaged heat balance equation are the components of vector $\vec{Q}_h$, the so-called *turbulent kinematic heat flux* (also called the turbulent temperature flux).

Turbulent fluxes of heat

This name becomes apparent if we consider products of these quantities, density, and c_p, *e.g.*,

$$\rho c_p \overline{u'\theta'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg\ K}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\mathrm{K}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

Turbulent fluxes of heat

$$\rho c_p \overline{u^{'}\theta^{'}} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg\ K}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\mathrm{K}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

The above term, for instance, has a meaning of heat energy transported, in average, by turbulent fluctuations $u^{'}$ (*i.e.*, in the x direction) per unit time per unit area of the surface normal to the x axis.

Turbulent fluxes of heat

$$\rho c_p \overline{u'\theta'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg\ K}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\mathrm{K}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

These terms are also known as the sensible heat flux, whose components are given by

- $Q_{h_x} = \rho c_p \overline{u'\theta'}$
- $Q_{h_y} = \rho c_p \overline{v'\theta'}$
- $Q_{h_z} = \rho c_p \overline{w'\theta'}$

Turbulent fluxes of heat

$$\rho c_p \overline{u'\theta'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg\ K}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\mathrm{K}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

Note, that in meteorological literature, the turbulent heat flux is usually normalized by density and c_p (*i.e.*, $\overline{u'\theta'} = Q_{h_x}/(\rho c_p)$).

These normalized quantities are often called *turbulent heat flux* with the word *kinematic* being omitted.

Turbulent fluxes of heat

We now apply the notion of turbulent heat flux in the Reynolds-averaged momentum balance equations

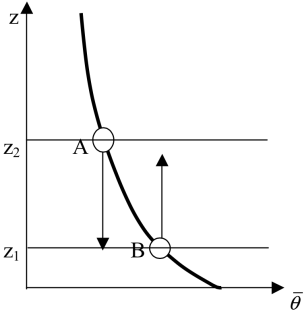
$$\frac{\partial \overline{\theta}}{\partial t} + \overline{u}\frac{\partial \overline{\theta}}{\partial x} + \overline{v}\frac{\partial \overline{\theta}}{\partial y} + \overline{w}\frac{\partial \overline{\theta}}{\partial z} + \frac{1}{\overline{\rho}\ c_p}\left(\frac{\partial Q_{h_x}}{\partial x} + \frac{\partial Q_{h_y}}{\partial y} + \frac{\partial Q_{h_z}}{\partial z}\right) + \overline{S_\theta} = 0$$

Turbulent fluxes of heat

$$\frac{\partial\overline{\theta}}{\partial t}+\overline{u}\frac{\partial\overline{\theta}}{\partial x}+\overline{v}\frac{\partial\overline{\theta}}{\partial y}+\overline{w}\frac{\partial\overline{\theta}}{\partial z}+\frac{1}{\overline{\rho}\ c_p}\left(\frac{\partial Q_{h_x}}{\partial x}+\frac{\partial Q_{h_y}}{\partial y}+\frac{\partial Q_{h_z}}{\partial z}\right)+\overline{S_\theta}=0$$

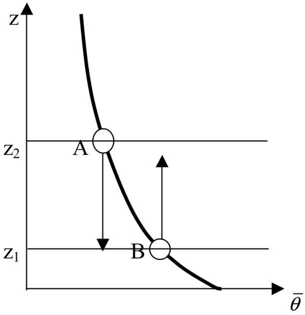
We have neglected diffusion terms because divergences of molecular heat fluxes under typical atmospheric boundary layer flow conditions are considerably smaller than their turbulent counterparts.

Turbulent fluxes of heat



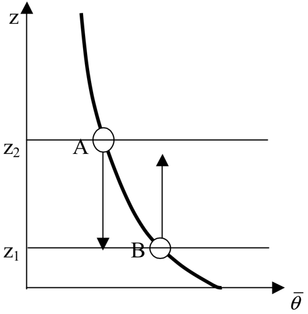
Suppose we have an idealized turbulent eddy near the ground on a hot summer day. If we start with a particular profile of θ, how will it change with time?

Turbulent fluxes of heat



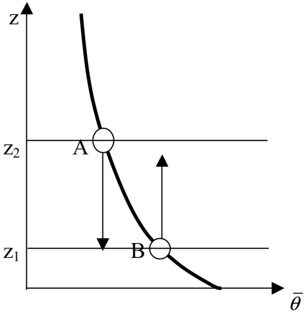
Due to surface heating, θ is typically super-adiabatic near the ground ($\partial\overline{\theta}/\partial z < 0$),

Turbulent fluxes of heat



Assume that we have two parcels, A and B. Parcel A moves downward and parcel B upward. When A moves downward, it becomes colder than its environment. Accordingly, it carries a negative θ'. Parcel B, on the other hand, becomes warmer than its environment and thus carries a positive θ'.

Turbulent fluxes of heat



Therefore
Parcel A: $w' < 0$ and $\theta' < 0 \rightarrow \overline{w'\theta'} > 0$ (positive heat flux)
Parcel B: $w' > 0$ and $\theta' > 0 \rightarrow \overline{w'\theta'} > 0$ (positive heat flux)

The End