Mountain Forced Flows

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Overview

Mountain Waves
- Trapped Lee Waves
- Lee Waves and Rotors
- Nonlinear Flows Over Two-Dimensional Mountains
- Generation of Severe Downslope Winds

Trapped Lee Waves

One of the most prominent features of mountain waves is the long train of wave clouds over the lee of mountain ridges in the lower atmosphere. This type of wave differs from the dispersive tails shown earlier for the $l \approx k$ case in that it is located in the lower atmosphere and there is no vertical phase tilt.

These types of trapped lee waves occur when the Scorer parameter decreases rapidly with height (Scorer 1949).

Trapped Lee Waves

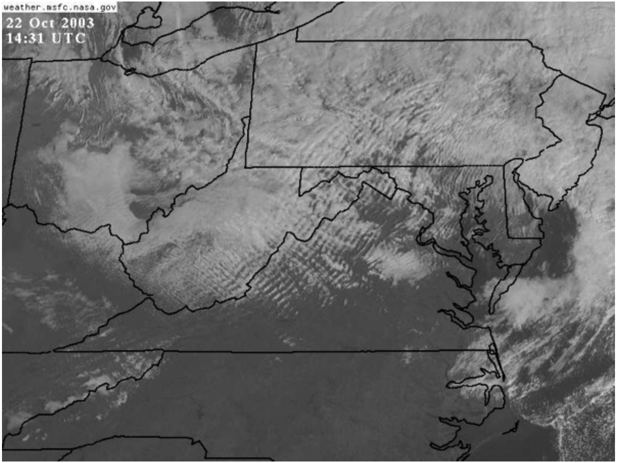


Figure: Satellite imagery for lee wave clouds observed at 1431 UTC, 22 October 2003, over western Virginia. Clouds originate at the Appalachian Mountains. (Courtesy of NASA).

Trapped Lee Waves

In considering flows forced by two-dimensional sinusoidal mountains and two-dimensional isolated mountains, we made the following assumptions and approximations when constructing the equations of motion:

- steady-state ($\partial/\partial t = 0$)
- non-rotating ($f = 0$)
- adiabatic $\dot{q} = 0$
- Boussinesq (incompressible, $\nabla\vec{u} = 0$; $\rho = \overline{\rho}$, except with g)
- inviscid (F_{rx} and $F_{rz} = 0$)
- two-dimensional ($v = 0$, $\partial/\partial y = 0$)
- $\overline{u}(z) =$ constant
- $N(z) =$ constant

Trapped Lee Waves

Here, we modify these assumptions to allow for vertical variation in either the zonal wind or static stability

i.e., $\overline{u}(z)$ or $N(z) \neq$ constant.

Here, the governing equation is

$$\nabla^2 w' + l^2(z) w' = 0 \tag{1}$$

Trapped Lee Waves

The Scorer parameter takes its full form

$$l^2(z) = \frac{N^2(z)}{\overline{u}^2(z)} - \frac{1}{\overline{u}(z)} \frac{\partial^2 \overline{u}(z)}{\partial z^2} \ . \tag{2}$$

The condition for vertical propagation becomes $k_s < l$, where k_s is the s-th Fourier component of the topography.

Trapped Lee Waves

If the mean cross mountain wind speed increases strongly with height

or

there is a low-level stable layer so that N decreases strongly with height

there may two layers of fluid with different Scorer parameter values

- $l_U \rightarrow$ upper layer
- $l_L \rightarrow$ lower layer

Trapped Lee Waves

Assume $l_U < l_L$

This means that waves whose wavenumbers fall between the two Scorer parameter values will:

- propagate vertically in the lower layer
- decay with height in the upper layer

Trapped Lee Waves

When the zonal wind or static stability is approximately constant within each layer, waves can experience refraction and/or reflection at the adjoining interface.

This is physically similar to optical rays passing through fluids of varying density.

Trapped Lee Waves

For reflection, each upward-propagating wave has an associated downward-propagating wave. These waves interact with one another.

The interaction of these waves is constructive (destructive) when the amplitude increases (decreases).

Trapped Lee Waves

For constructive interference, lee wave energy becomes trapped in the lower layer, which is then called a *wave duct*.

The trapped wave is capable of transporting energy over long distances with little attenuation.

Trapped Lee Waves

Linear theory is expected to reproduce accurate results when the *non-dimensional mountain height* $Nh_m/\overline{u} \ll 1$.

Even when this value is not necessarily small, in the cases with constant $\overline{u}$ and N, the difference between linear predictions and the full non-linear solutions remains small so long that waves do not break.

Trapped Lee Waves

However, these differences become dramatic when $\overline{u}$ and N vary in a way that produces trapped lee waves.

Numerical simulations have shown that linear theory only reliably predicts wave amplitude when the ratio of lee wavelength to mountain width is greater than unity.

Trapped Lee Waves

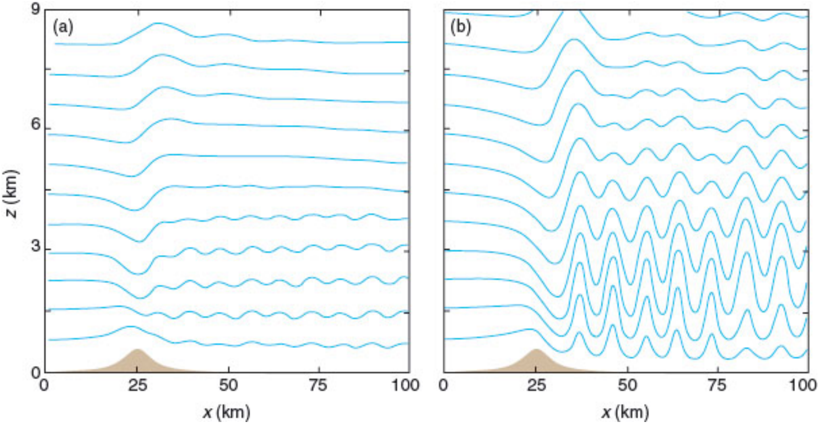


Figure: Streamlines in air flow over a mountain for (a) steady flow subject to the linear approximation and (b) the fully nonlinear and unsteady solution. [From Markowski and Richardson]

Lee Waves and Rotors

Due to the co-existence of the upward propagating waves and downward propagating waves, there exists no phase tilt in the lee waves.

Once lee waves form, regions of reversed cross-mountain winds near the surface beneath the crests of the lee waves may develop due to the presence of a reversed pressure gradient force.

Lee Waves and Rotors

In the presence of surface friction, a sheet of vorticity parallel to the mountain range forms along the lee slopes, which originates in the region of high shear within the boundary layer.

The vortex sheet separates from the surface, ascends into the crest of the first lee wave, and remains aloft as it is advected downstream by the undulating flow in the lee waves

Lee Waves and Rotors

The vortex with recirculated air is known as rotor and the process that forms it is known as boundary layer separation.

These rotors are often observed to the lee of steep mountain ranges such as over the Owens Valley, California, on the eastern slope of Sierra Nevada).

Occasionally, a turbulent, altocumulus cloud forms with the rotor and is referred to as rotor cloud.

Lee Waves and Rotors

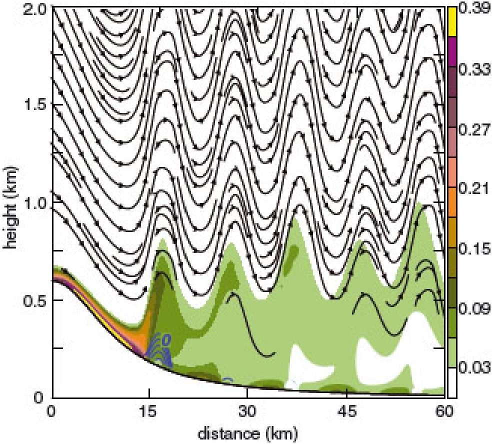


Figure: Streamlines and horizontal vorticity (colored, s^{-1}) in a numerical simulation using a no-slip lower-boundary condition. Horizontal wind speeds less than or equal to zero are shown using blue isotachs (every $2\ \mathrm{ms}^{-1}$). [From Markowski and Richardson]

Nonlinear Flows Over Two-Dimensional Mountains

The linear dynamics of mountain waves over 2D ridges are fundamentally understood.

Linear theory, however, begins to break down when the perturbation velocity ($u^{'}$) becomes large compared with the basic flow ($\overline{u}$) in some regions, so that the flow becomes stagnant.

Nonlinear Flows Over Two-Dimensional Mountains

This happens when the mountain becomes very high, the basic flow becomes very slow, or the stratification becomes very strong.

In other words, flow becomes more nonlinear when the *Froude number*, $\overline{u}/Nh_m$, becomes small.

Nonlinear Flows Over Two-Dimensional Mountains

Thus, in order to fully understand the dynamics of nonlinear phenomena, such as upstream blocking, wave breaking, severe downslope winds and lee vortices, we need to take a nonlinear approach.

Nonlinear response of a continuously stratified flow over a mountain is very complicated since the nonlinearity may come from the basic flow characteristics, the mountain height, or the transient behavior of the internal flow, such as wave steepening.

Nonlinear Flows Over Two-Dimensional Mountains

Another way to understand why linear theory fails in these cases:

In linear theory, the forcing at a particular wavelength is obtained by the Fourier transform of the mountain.

As a result, little forcing is produced at the resonant wavelength if the mountain is much wider than that wavelength.

Nonlinear Flows Over Two-Dimensional Mountains

The non-linear wave amplitude when resonant wavelengths are short compared to the mountain width is due to enhancement of shorter wavelengths through nonlinear wave interactions rather than through direct terrain forcing.

These interactions are not accounted for in linear theory.

Nonlinear Flows Over Two-Dimensional Mountains

Long (1955) derived the governing equation for the finite-amplitude, steady state, two-dimensional, inviscid, continuously and stably stratified flow.

He obtained an equation for vertical displacement of a streamline from its far upstream δ. It looks essentially the same as the equation for w' that we derived earlier for linear waves:

$$\nabla^2\delta + l^2\delta = 0 \ , \qquad (3)$$

where $l = N/\overline{u}$ is the Scorer parameter of the basic flow far upstream.

Nonlinear Flows Over Two-Dimensional Mountains

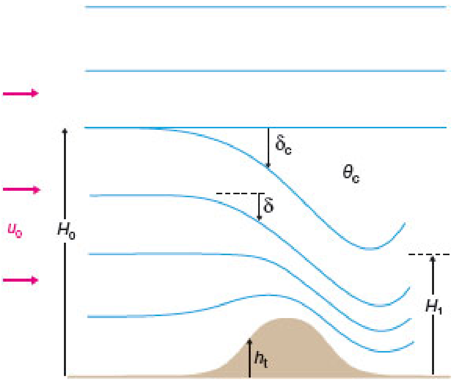


Figure: A certain critical streamline divides and encompasses a region of uniform potential temperature. H_0 is the original height of the dividing streamline, θ_c is the potential temperature in the well-mixed region between the split streamlines, δ is the displacement of an arbitrary streamline, δ is the displacement of the dividing streamline, and H_1 is the nadir of the lower dividing streamline. [From Markowski and Richardson]

Nonlinear Flows Over Two-Dimensional Mountains

The main difference from the linear problem is that non-linear lower boundary condition has to be used to represent the finite-amplitude mountain:

$$\delta(x, z) = h(x) \quad \text{at } z = h(x) \ . \tag{4}$$

The nonlinear lower boundary condition is applied on the mountain surface, instead of approximately applied at $z = 0$ as in the linear lower boundary condition.

Nonlinear Flows Over Two-Dimensional Mountains

In the next figure, streamlines of analytical solutions are shown for flow over a semi-circle obstacle for the non-dimensional mountain heights $= 0.5$, 1.0, 1.27, and 1.5.

The non-dimensional mountain height (the reciprocal of the Froude number) is a measure of the non-linearity of the continuously stratified flow.

Nonlinear Flows Over Two-Dimensional Mountains

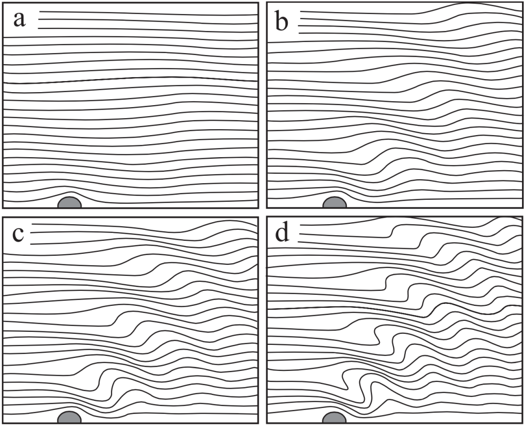


Figure: Streamlines of Longs model solutions for uniform flow over a semi-circle obstacle with $Nh/U =$ (a) 0.5, (b) 1.0, (c) 1.27, and (d) 1.5. Note that the streamlines become vertical in (c) and overturn in (d). (Adapted after Miles 1968)

Nonlinear Flows Over Two-Dimensional Mountains

When $Nh_m/\overline{u}$ is small (*e.g.*, 0.5) the flow is more linear.

As $Nh_m/\overline{u}$ increases to 1.27, the flow becomes more non-linear and its streamlines become vertical at the first level of wave steepening.

For flow with $Nh_m/\overline{u} > 1.27$, the flow becomes statically and dynamically (shear) unstable.

Generation of Severe Downslope Winds

Severe downslope winds over the lee of a mountain ridge have been observed in various places around the world, such as the *chinook* over the Rocky Mountains, *foehn* over the Alps.

One well-known event is the 11 January 1972 windstorm that occurred in Boulder, Colorado.

With this event, the peak wind gust reached as high as 60 ms^{-1} (!!!) and produced severe damage in the Boulder, Colorado area.

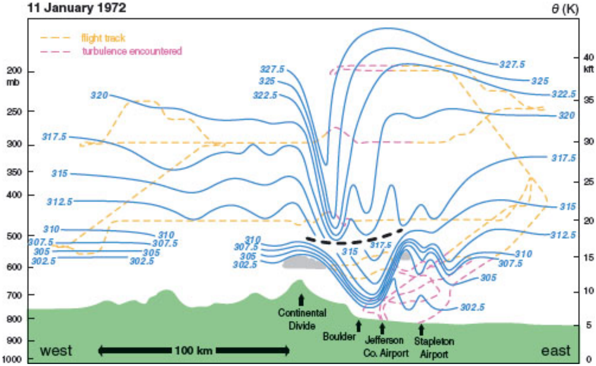


Figure: Analysis of potential temperatures (blue contours; K) from aircraft flight data and rawinsondes on 11 January 1972 during a downslope windstorm near Boulder, CO. [From Markowski and Richardson]

Generation of Severe Downslope Winds

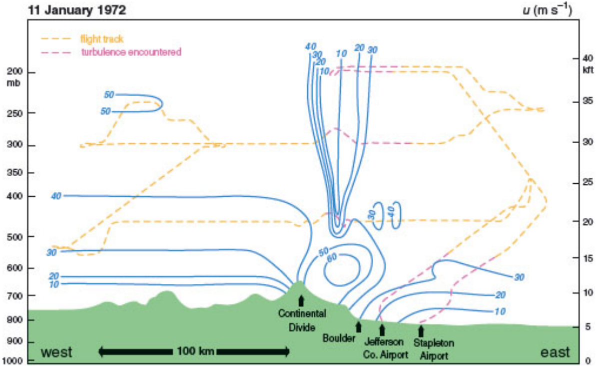


Figure: Analysis of the westerly wind component (blue contours; m/s) on 11 January 1972 during the downslope windstorm near Boulder, CO. [From Markowski and Richardson]

Generation of Severe Downslope Winds

The basic dynamics of the severe downslope wind can be understood from the following two major theories:

(a) *resonant amplification theory*

(b) *hydraulic jump theory*

(along with later studies on the effects of instabilities, wave ducting, nonlinearity, and upstream flow blocking)

Resonant Amplification Theory

Idealized nonlinear numerical experiments indicate that a high-drag (severe-wind) state occurs after an upward propagating mountain wave breaks above a mountain.

The wave-breaking region is characterized by strong turbulent mixing, with a local wind reversal on top of it.

The wind reversal level coincides with the critical level for a stationary mountain wave and, thus, is also referred to as the *wave-induced critical level*.

Resonant Amplification Theory

Waves can not propagate through the critical level and are reflected downwards.

The wave breaking region aloft acts as an internal boundary which reflects the upward propagating waves back to the ground and produces a high-drag state through partial resonance with the upward propagating mountain waves.

Resonant Amplification Theory

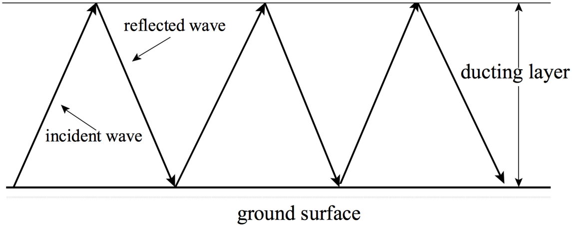


Figure: Illustration of wave reflection and transmission between the ground surface and some upper level. [From Nappo, 2002]

Resonant Amplification Theory

When the critical level is located at a non-dimensional height of $z_i/\lambda_z = 3/4 + n$ above the surface, non-linear resonant amplification occurs between the upward propagating waves generated by the mountain and the downward propagating waves reflected from the critical level. (n is an integer, z_i is a prescribed critical level height, $\lambda_z = 2\pi\overline{u}/N$ is the vertical hydrostatic length scale)

On the other hand, if the basic-flow critical level is located at a non-dimensional height $z_i/\lambda_z \neq 3/4 + n$ (*e.g.*, 1.15), there is no wave resonance and no severe downslope winds are generated.

Resonant Amplification Theory

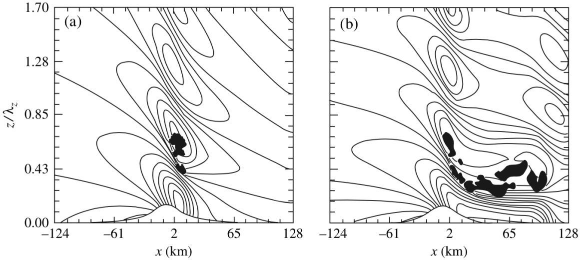


Figure: Wave ducting as revealed by the time evolution of horizontal wind speeds and regions of local $Ri < 0.25$ (shaded) for a flow with uniform wind and constant static stability over a mountain ridge at ut/a (a) 12.6, and (d) 50.4. The Froude number of the uniform basic wind is 1.0. (Adapted after Wang and Lin 1999)

Resonant Amplification Theory

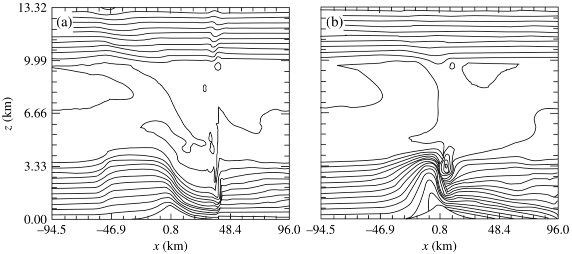
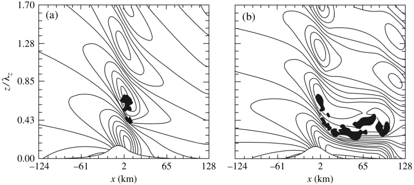
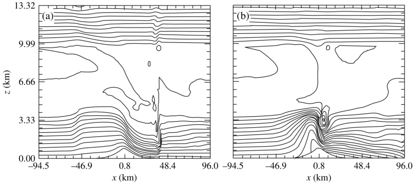


Figure: Effects of nonlinearity on the development of severe downslope winds: (a) Potential temperature field from nonlinear numerical simulations for a basic flow with and ; (b) Same as (a) except from linear numerical simulations. The contour interval is 1 K in both (a) and (b). (Adapted after Wang and Lin 1999)

Resonant Amplification Theory

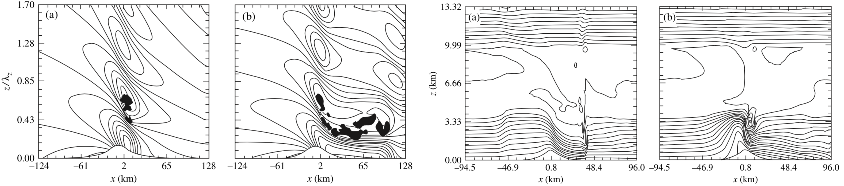




Shortly after the occurrence of wave breaking, regions with local $Ri < 0.25$ form

This turbulent mixing region expands downward and downstream due to strong non-linear effects on the flow near the critical level

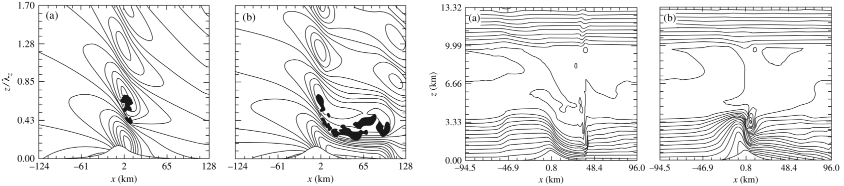
Resonant Amplification Theory



This region expands downward by wave reflection and ducting from the wave-induced critical level and accelerates downstream by the non-linear advection.

Effects of wave reflection are evidenced by the fact that the wave duct with severe downslope wind is located below the region of the turbulent mixing.

Resonant Amplification Theory



In the absence of non-linearity, the wave-breaking region does not expand downward to reduce the depth of the lower uniform wind layer.

This, in turn, prohibits the formation of the severe downslope wind and internal hydraulic jump.

Hydraulic Jump Theory

Hydraulic Jump Theory

The above image depicts a hydraulic jump in a kitchen sink, which shows a very rapid change in the flow depth across the jump. Around the place where the tap water hits the sink, you will see a smooth looking flow pattern. A little further away, you will see a sudden ‘jump’ in the water level. This is a hydraulic jump.

Hydraulic Jump Theory

A hydraulic theory was proposed to explain the development of severe downslope winds based on the similarity of flow configurations of severe downslope windstorms and finite-depth, homogeneous flow over a mountain ridge. (Smith 1985)

Hydraulic Jump Theory

The hydraulic theory attributes the high-drag (severe-wind) state to the interaction between a smoothly stratified flow and the deep, well-mixed, turbulent dead region above the lee slope in the middle troposphere.

Though not directly applicable to stratified atmosphere associated with downslope wind storms, using shallow water theory will offer some intuition about flow traversing a barrier.

Math Break

Pages 10 and 11 of the notes.

Hydraulic Jump Theory

$\mathbf{Fr > 1}$

- On the windward side, $\partial h_t/\partial x > 0$ and $\partial D/\partial x > 0$.
- Thus, fluid traveling uphill will thicken, reaching its maximum thickness at the peak of the mountain.
- On the lee side, $\partial h_t/\partial x < 0$ and $\partial D/\partial x < 0$
- Thus, fluid traveling downhill will thin.
- As the thickness of the fluid changes, the zonal velocity also changes to maintain constant mass flux.
- Thus, when $\mathrm{Fr} > 1$, there is a minimum in zonal velocity at the top of the mountain where the fluid is thickest.

$\mathbf{Fr > 1}$

If we consider a parcel embedded in a westerly wind, the parcel will decelerate as it passes over the mountain and then return to its original wind speed at the leeward base of the mountain (assuming we neglect friction).

This regime represents a transfer of energy from kinetic to potential and back to kinetic. We find such behavior to be quite intuitive.

Consider, for example, a ball rolling up a hill then descending once it crests the peak. Flow where $\mathrm{Fr} > 1$ is called *supercritical flow*.

Hydraulic Jump Theory

$\mathbf{Fr} < \mathbf{1}$

- On the windward side, $\partial h_t / \partial x > 0$ and $\partial D / \partial x < 0$.
- Thus, fluid traveling uphill will thin, reaching its maximum thinness at the peak of the mountain.
- On the lee side, $\partial h_t / \partial x < 0$ and $\partial D / \partial x > 0$
- Thus, luid traveling downhill will thicken.

Hydraulic Jump Theory

$\mathbf{Fr < 1}$

The acceleration obtained by an isolated air parcel depends on the difference between the pressure gradient force arising from changes in the fluid depth versus the amount of work associated with ascending the terrain.

Here, the pressure gradient force dominates and leads to a net positive acceleration following the parcel as it ascends.

On the lee side, the fluid thickens and returns to its original depth as the parcel decelerates to its original speed. Flow where $\mathrm{Fr} < 1$ is called *subcritical flow*.

Hydraulic Jump Theory

$\mathbf{Fr \approx 1}$

Both of the previously considered cases both return the parcel to its original wind speed when it reaches the lee side.

How do we get winds that accelerate along the entire path from the windward to the leeward side?

For the windward side, this acceleration requires subcritical flow, while on the leeward side the acceleration requires supercritical flow.

In other words, the acceleration on the windward side must cause u to cross the threshold from subcritical to supercritical flow, which is likely to happen only if the flow has a Fr close to unity at the start.

Hydraulic Jump Theory

$\mathbf{Fr \approx 1}$

The transition from subcritical to supercritical results in leeward wind speeds that exceed their original value on the windward side.

In accordance with the increasing speeds, the fluid thickness decreases over the entire path.

This causes the free surface to drop sharply on the leeward side (analogous to the descending isentropes during downslope wind events) and results in what is called a hydraulic jump.

Hydraulic jumps are very turbulent, and large amounts of energy are dissipated within them.

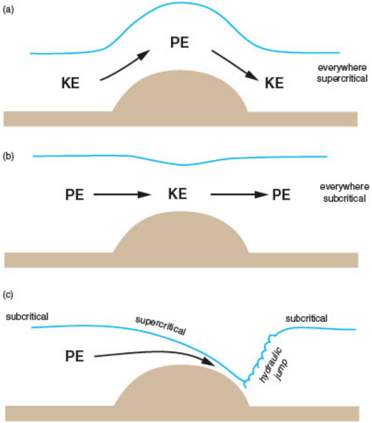


Figure: Flow over an obstacle for the simple case of a single layer of fluid having a free surface. (a) Supercritical flow ($\mathrm{Fr} > 1$) everywhere, (b) Subcritical flow ($\mathrm{Fr} < 1$) everywhere, (c) Supercritical flow on the lee slope with adjustment to subcritical flow at a hydraulic jump near the base of the obstacle. [From Markowski and Richardson])

The End