MESOSCALE METEOROLOGY

METR 4433 | Problem Set #1 | Spring 2015

Assigned: Feb. 3, Due: Feb 17

1.) Scale Analysis (10 points)

Using the appropriate horizontal and vertical momentum equations, perform a scale analysis for a mesoscale mountain disturbance and describe the main characteristics of motion at this scale (*i.e.*, what does each term tell you about the flow?). Hint: Assume Orlanski's definitions of mesoscale.

2.) Linear Perturbation Theory (10 points)

In Section 2.1.2, we developed a set of two-dimensional, irrotational, inviscid, and adiabatic equations of motion to describe internal atmospheric wave motions. In order to linearize the equations, we assumed the following base state:

$$
\begin{aligned}
u(t,x,z) &= \overline{u}(z) + u^{'}(t,x,z) \\
w(t,x,z) &= w^{'}(t,x,z) \\
p(t,x,z) &= \overline{p}(z) + p^{'}(t,x,z) \\
\theta(t,x,z) &= \overline{\theta}(z) + \theta^{'}(t,x,z) \\
\rho(t,x,z) &= \overline{\rho}(z) + \rho^{'}(t,x,z) \quad \text{if paired with } g \\
\rho(t,x,z) &= \overline{\rho}(z) \quad \text{otherwise}
\end{aligned}
$$

Use these base-state expressions to derive the linearized equations of motion given in the notes. That is, show:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u^{'}}{\partial t} + \overline{u}\frac{\partial u^{'}}{\partial x} + w^{'}\frac{\partial \overline{u}}{\partial z} + \frac{1}{\overline{\rho}}\frac{\partial p^{'}}{\partial x} = 0 \tag{1}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g = \frac{\partial w^{'}}{\partial t} + \overline{u}\frac{\partial w^{'}}{\partial x} + \frac{1}{\overline{\rho}}\frac{\partial p^{'}}{\partial z} - g\frac{\theta^{'}}{\overline{\theta}} = 0 \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \frac{\partial u^{'}}{\partial x} + \frac{\partial w^{'}}{\partial z} = 0 \tag{3}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = \frac{\partial \theta^{'}}{\partial t} + \overline{u}\frac{\partial \theta^{'}}{\partial x} + w^{'}\frac{N^2\overline{\theta}}{g} = 0\,, \tag{4}$$

where N^2 is the Brunt-Väisälä frequency.

Show your work and make sure to describe the steps along the way. For instance, the gravity term in Eq.(2) was simply given in the notes. You will need to actually prove it. Hints: See section 2.3.3 in Markowski or section 7.4.1 in Holton (show your work - don't just take their word for it). Remember to ignore non-linear terms.

3.) Flows Over Two-Dimensional Sinusoidal Mountains (15 points)

(a) Combine Eqs.(1)-(4) to derive Scorer's equation. Make sure to define the equation in terms of the Scorer parameter. Assume the flow is steady-state and that both the mean wind and static stability are constant with height.

(b) We found in class that the general solution to Scorer's equation for this flow is

$$w' = A\exp\left(i[kx + mz]\right) + B\exp\left(i[kx - mz]\right) ,$$

where $m = \sqrt{l^2 - k^2}$. Derive the two solutions for w' for the cases where $l > k$ and $l < k$, respectively. Describe the type of wave associated with each solution. Hint: The notes will be especially helpful.

(c) What type of atmospheric phenomenon occurs if we allow the mean wind and static stability to vary with height such that the Scorer parameter decreases rapidly with height?

4.) Blocking of Wind by Terrain (15 points)

In class, we derived the combined Bernoulli equation for a streamline approaching terrain

$$\Pi' + \overline{\Pi} + \frac{u^2}{2c_p\theta_0} + \frac{gz}{c_p\theta_0} = \Pi_0 + \frac{u_0^2}{2c_p\theta_0} + \frac{gz_0}{c_p\theta_0} ,$$

where the upstream value of $\overline{\Pi}$ is given by

$$\overline{\Pi}(z) = \Pi_0 - \frac{g\delta}{c_p\theta_0} + \frac{N^2\delta^2}{2c_p\theta_0} ,$$

and δ is the vertical displacement of a streamline as it approaches the terrain.

(a) Use these expressions to derive an equation that describes the speed of an air parcel (u^2) approaching the terrain. Assume that pressure perturbations are small enough to neglect.

(b) Using this equation, derive the maximum vertical displacement that is allowed before the flow stagnates.

(c) The mountain Froude number is defined as $\mathrm{F_{rm}} = u/(Nh_m)$, where h_m is the mountain height. Redefine $\mathrm{F_{rm}}$ in terms of the maximum vertical displacement. Using this mountain Froude number, describe what you would expect in terms of blocking for flow over the mountain if $\mathrm{F_{rm}} > 1$ and $\mathrm{F_{rm}} < 1$. Be sure to relate the equation to a physical reason for each case.

(d) What is the main problem with using this approach to assess blocking potential?