Mesoscale Meteorology

METR 4433 Spring 2015

3.3.2 Reynolds Fluxes and Their Physical Interpretation

Turbulent flux of momentum and turbulent stress

The variances and covariances ($\overline{u'u'}$, $\overline{u'v'}$, $\overline{u'w'}$, $\overline{v'u'}$, $\overline{v'v'}$, $\overline{v'w'}$, $\overline{w'u'}$, $\overline{w'v'}$, $\overline{w'w'}$) that appear in the Reynolds-averaged Navier-Stokes equations are the components of the *turbulent kinematic momentum flux*. This name becomes apparent if we consider products of these quantities and density, *e.g.,*

$$\rho\overline{w'u'} \rightarrow \left[\frac{\text{kg}}{\text{m}^3}\right] \left[\frac{\text{m}}{\text{s}}\right] \left[\frac{\text{m}}{\text{s}}\right] = \left[\frac{\text{kg ms}^{-1}}{\text{m}^2\ \text{s}}\right] = \left[\frac{\text{kg m}}{\text{m}^2\ \text{s}^2}\right] = \left[\frac{\text{N}}{\text{m}^2}\right].$$

The above term, for instance, has a meaning of x component of momentum transported in average by fluctuating velocity component $w^{'}$ per unit time per unit area of the surface normal to z axis. On the other hand, this term may also be interpreted as z component of momentum transported in average by fluctuating velocity component $u^{'}$ per unit time per unit area of the surface normal to x axis. Thus, these fluxes can be thought of as the transport of mass per unit area per unit time. In other words, they represent the force per unit area.

Physical quantities having opposite signs to the momentum flux components are components of the turbulent stress:

- $\tau_{xx} = -\rho\overline{u'u'}$
- $\tau_{xy} = -\rho\overline{u'v'}$
- $\tau_{xz} = -\rho\overline{u'w'}$
- $\tau_{yx} = -\rho\overline{v'u'}$
- $\tau_{yy} = -\rho\overline{v'v'}$
- $\tau_{yz} = -\rho\overline{v'w'}$
- $\tau_{zx} = -\rho\overline{w'u'}$
- $\tau_{zy} = -\rho\overline{w'v'}$
- $\tau_{zz} = -\rho\overline{w'w'}$,

where $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$. Here, τ_{xx}, τ_{yy}, τ_{yy} are the normal components of the turbulent stress, while $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$ are the shear components of the stress. Why is this called a stress? Why do we describe the components as normal and shear?

Consider an idealized cubic volume.

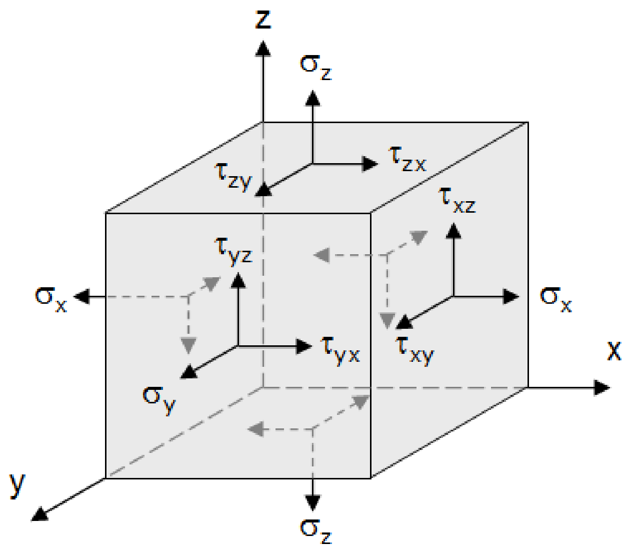

Figure 1: Idealized cubic volume indicating turbulent stresses. Note here, *e.g.*, $\sigma_x = \tau_{xx}$.

Let's consider our previous example of τ_{xz}. Here, the flux of the momentum parallel to a volume face (*e.g.*, the flux of $u^{'}$ through lower boundary of a cubed volume by $w^{'}$) causes the parallel momentum ($u^{'}$ in this case) to change at that face. Accordingly, the momentum flux is the *stress* applied at this face. We therefore call the momentum fluxes in the momentum equations the *Reynolds stresses*.

Note, that in meteorological literature, the turbulent momentum flux and turbulent shear stress are usually normalized by density (*i.e.*, -$\overline{w^{'}u^{'}} = \tau_{zx}/\rho$). These normalized quantities are often called *turbulent momentum flux* and *turbulent shear stress* with the word *kinematic* being omitted.

We now apply the notion of turbulent stresses and momentum flux in the Reynolds-averaged momentum balance equations

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} + \overline{w}\frac{\partial \overline{u}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x} - f\overline{v} = 0 \tag{1}$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + \overline{w}\frac{\partial \overline{v}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial y} + f\overline{u} = 0 \tag{2}$$

$$\frac{\partial \overline{w}}{\partial t} + \overline{u}\frac{\partial \overline{w}}{\partial x} + \overline{v}\frac{\partial \overline{w}}{\partial y} + \overline{w}\frac{\partial \overline{w}}{\partial z} - \frac{1}{\overline{\rho}}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial z} + g - \beta\overline{\theta^{'}} = 0\,. \tag{3}$$

In models of atmospheric flows (including boundary-layer flows), which are essentially turbulent (with the exception of some special flow cases), viscous terms on the right-hand sides of our original equations are usually neglected. This is a reasonably safe assumption in turbulent boundary layer flows since the effects of molecular diffusion are much smaller than the effects of turbulent eddies.

Turbulent fluxes of heat

The covariances ($\overline{u'\theta'}$, $\overline{v'\theta'}$, and $\overline{w'\theta'}$) that appear in the Reynolds-averaged heat balance equation are the components of vector $\vec{Q_h}$, the so-called *turbulent kinematic heat flux* (also called the turbulent temperature flux). This name becomes apparent if we consider products of these quantities, density, and c_p, *e.g.*,

$$\rho c_p \overline{u'\theta'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg\ K}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\mathrm{K}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\ \mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

The above term, for instance, has a meaning of heat energy transported, in average, by turbulent fluctuations $u^{'}$ (*i.e.*, in the x direction) per unit time per unit area of the surface normal to the *x* axis. These terms are also known as the sensible heat flux, whose components are given by

- $Q_{h_x} = \rho c_p \overline{u'\theta'}$
- $Q_{h_y} = \rho c_p \overline{v'\theta'}$
- $Q_{h_z} = \rho c_p \overline{w'\theta'}$

Note, that in meteorological literature, the turbulent heat flux is usually normalized by density and c_p (*i.e.*, $\overline{u'\theta'} = Q_{h_x}/(\rho c_p)$. These normalized quantities are often called *turbulent heat flux* with the word *kinematic* being omitted. We now apply the notion of turbulent heat flux in the Reynolds-averaged momentum balance equations

$$\frac{\partial \overline{\theta}}{\partial t} + \overline{u}\frac{\partial \overline{\theta}}{\partial x} + \overline{v}\frac{\partial \overline{\theta}}{\partial y} + \overline{w}\frac{\partial \overline{\theta}}{\partial z} + \frac{1}{\overline{\rho}\, c_p}\left(\frac{\partial Q_{h_x}}{\partial x} + \frac{\partial Q_{h_y}}{\partial y} + \frac{\partial Q_{h_z}}{\partial z}\right) + \overline{S_\theta} = 0 \tag{4}$$

We have neglected diffusion terms because divergences of molecular heat fluxes under typical atmospheric boundary layer flow conditions are considerably smaller than their turbulent counterparts.

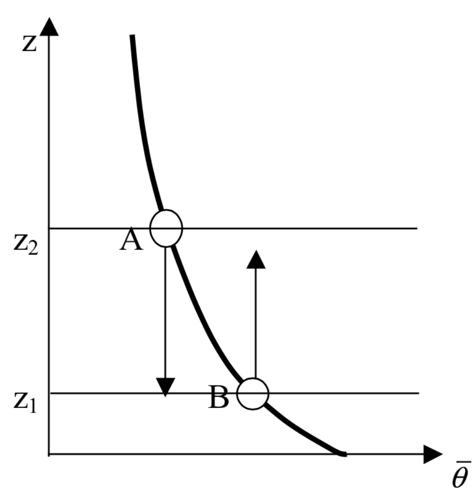

Figure 2: Idealized depiction of two air parcel being transported by turbulent eddies.

Let's look at the physical meaning of these fluxes in more detail.

Suppose we have an idealized turbulent eddy near the ground on a hot summer day. If we start with a particular profile of θ, how will it change with time?

Due to surface heating, θ is typically super-adiabatic near the ground ($\partial\overline{\theta}/\partial z < 0$), as shown here in Fig. 2.

Assume that we have two parcels, A and B. Parcel A moves downward and parcel B upward. When A moves downward, it becomes colder than its environment. Accordingly, it carries a negative $\theta^{'}$. Parcel B, on the other hand, becomes warmer than its environment and thus carries a positive $\theta^{'}$.

Therefore:

- Parcel A
  - $w^{'} < 0$ and $\theta^{'} < 0 \rightarrow \overline{w'\theta'} > 0$ (positive heat flux)
- Parcel B
  - $w^{'} > 0$ and $\theta^{'} > 0 \rightarrow \overline{w'\theta'} > 0$ (positive heat flux)

Even though both fluxes are positive, the physical process is different. Positive heat flux can be caused by either transporting (by turbulent eddies) colder air downward or transporting warmer air upward.

If we only consider the vertical turbulent heat flux (which tend to dominate in convective boundary layer), assume that the mean velocity is zero (*i.e.*, there is no mean wind advection), and neglect molecular diffusion effects, then the Reynolds-averaged thermodynamic energy equation becomes

$$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z}\,. \tag{5}$$

We see that temperature changes as a result of the *heat flux divergence*, not the heat flux itself. Thus, temperature changes only when the net heat flux into an air parcel (or volume) is non-zero. In the above example, the flux at levels z_1 and z_2 are both positive. However, because the vertical gradient of θ is larger at z_1, the flux there has a larger magnitude. Meaning,

$$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z} = -\frac{\overline{w'\theta'}_{z_2} - \overline{w'\theta'}_{z_1}}{z_2 - z_1} > 0\,. \tag{6}$$

Therefore, the mean potential temperature θ in the layer between z_1 and z_2 will increase with time! If the heat flux is positive at both levels and increases with height, then the temperature will decrease in time (even though the heat flux is positive). This happens because more heat is leaving the layer at the top boundary that is entering from the bottom.

Similar reasoning can be applied to turbulent fluxes of other material quantities such as water vapor mixing ratio, and to momentum.

Turbulent fluxes of moisture

The covariances ($\overline{u'q'}$, $\overline{v'q'}$, and $\overline{w'q'}$) that appear in the Reynolds-averaged moisture balance equation are the components of vector $\vec{Q}_e$, the so-called *kinematic flux of water vapor* (also called the kinematic flux of moisture). This name becomes apparent if we consider products of these quantities, density, and L_v, *e.g.*,

$$\rho L_v \overline{u'q'} \rightarrow \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right] \left[\frac{\mathrm{J}}{\mathrm{kg}}\right] \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \left[\frac{\mathrm{kg}}{\mathrm{kg}}\right] = \left[\frac{\mathrm{J}}{\mathrm{m}^2\,\mathrm{s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right].$$

The above term, for instance, has a meaning of moisture energy transported, in average, by turbulent fluctuations $u^{'}$ (*i.e.*, in the x direction) per unit time per unit area of the surface normal to the x axis. These terms are also known as the latent heat flux, whose components are given by

- $Q_{e_x} = \rho L_v \overline{u'q'}$
- $Q_{e_y} = \rho L_v \overline{v'q'}$
- $Q_{e_z} = \rho L_v \overline{w'q'}$.

Note, that in meteorological literature, the turbulent moisture flux is usually normalized by density and L_v (*i.e.*, $\overline{u'q'} = Q_{e_x}/(\rho L_v)$. These normalized quantities are often called *turbulent moisture flux* with the word *kinematic* being omitted.

We now apply the notion of turbulent moisture flux in the Reynolds-averaged momentum balance equations

$$\frac{\partial \overline{q}}{\partial t} + \overline{u}\frac{\partial \overline{q}}{\partial x} + \overline{v}\frac{\partial \overline{q}}{\partial y} + \overline{w}\frac{\partial \overline{q}}{\partial z} + \frac{1}{\overline{\rho}\,L_v}\left(\frac{\partial Q_{e_x}}{\partial x} + \frac{\partial Q_{e_y}}{\partial y} + \frac{\partial Q_{e_z}}{\partial z}\right) + \overline{S_q} = 0 \tag{7}$$

We have again neglected diffusion terms because divergences of molecular moisture fluxes under typical atmospheric boundary layer flow conditions are considerably smaller than their turbulent counterparts.

Note, however, that molecular diffusion terms may become important for the flows in the very close vicinity of the underlying surface or during microscale physical processes (*e.g.*, in clouds).

3.3.3 Turbulence Closure Problem

As we have seen, the process of Reynolds averaging results in 12 additional terms - the turbulent Reynolds fluxes [note here that there are 12 and not 15 since $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, and $\tau_{yz} = \tau_{zy}$]. This means we need 12 new prognostic equations to solve for these second-order fluxes. As it turns out, the prognostic equations for these second-order fluxes contain third-order moments. Likewise, the prognostic equations for the third-order moments contain fourth-order moments - and so on. This is known as the *turbulence closure problem*. It means that we need a way to close our system of equations.

Parameterization of turbulent fluxes

A popular way to "solve" the turbulence closure problem is to make use of parameterizations - that is, relating unknown quantities to known terms. Fully considering the different parameterization schemes is beyond the scope of this course. However, we will briefly discuss one simple approach.

It is often assumed that turbulent eddies act in a way analogous to molecular diffusion. That is, the turbulent flux is assumed to be proportional to the local gradient of the corresponding mean value. Specifically, this is given by, *e.g.*,

$$\overline{u'w'} = -K_m \frac{\partial \overline{u}}{\partial z} \tag{8}$$

$$\overline{v'w'} = -K_m \frac{\partial \overline{v}}{\partial z} \tag{9}$$

$$\overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z} \tag{10}$$

$$\overline{w'q'} = -K_e \frac{\partial \overline{q}}{\partial z} \,. \tag{11}$$

where K_m, K_h, and K_e are called the *eddy viscosity*, *eddy diffusivity for heat*, and *eddy diffusivity for moisture*, respectively. This approach is often referred to as K-theory, K-closure, or *flux-gradient theory*. In this approach, the Reynolds-averaged equations give expressions for the mean variables, but we must prescribe the K terms. In the simplest case, these terms (sometimes called *transfer coefficients*) are taken as constant. In more complex schemes, K_m is parameterized in terms of the mean flow, static stability, and/or turbulence kinetic energy, while K_h and K_e are subsequently prescribed as a function of K_m. We will not cover such approaches.

Turbulent kinetic energy (TKE), $\overline{e}$, is a measure of the intensity of turbulence and is defined as

$$\overline{e} = \frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right) . \tag{12}$$

TKE is a maximum during the early afternoon in the middle of the boundary layer, and a minimum during the nighttime hours (see Fig. 3).

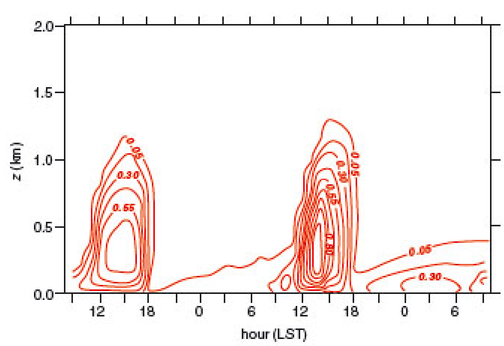

Figure 3: Simulation of the time and space variation of TKE ($\mathrm{m}^2/\mathrm{s}^2$) from the Wangara, Australia, boundary layer field experiment. [From Markowski and Richardson]

The procedure to obtain a prognostic equation for TKE is described in detail in Markowski and Richardson. We will not worry here about the details. The equation is given by

$$
\begin{aligned}
\underbrace{\frac{\partial \overline{e}}{\partial t}}_{(I)} &= \underbrace{-\overline{u}\frac{\partial \overline{e}}{\partial x} - \overline{v}\frac{\partial \overline{e}}{\partial y} - \overline{w}\frac{\partial \overline{e}}{\partial z}}_{(II)} + \underbrace{\frac{g}{\overline{\theta}}\overline{w'\theta'}}_{(III)} \\
& \underbrace{\begin{aligned} & - \overline{u'u'}\frac{\partial \overline{u}}{\partial x} - \overline{v'u'}\frac{\partial \overline{v}}{\partial x} - \overline{w'u'}\frac{\partial \overline{w}}{\partial x} - \overline{u'v'}\frac{\partial \overline{u}}{\partial y} \\ & - \overline{v'v'}\frac{\partial \overline{v}}{\partial y} - \overline{w'v'}\frac{\partial \overline{w}}{\partial z} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \overline{v'w'}\frac{\partial \overline{v}}{\partial z} - \overline{w'w'}\frac{\partial \overline{w}}{\partial z} \end{aligned}}_{(IV)} \\
& \underbrace{- \frac{\partial \overline{u'e}}{\partial x} - \frac{\partial \overline{v'e}}{\partial y} - \frac{\partial \overline{w'e}}{\partial z}}_{(V)} \\
& \underbrace{- \frac{1}{\overline{\rho}}\frac{\partial \overline{u'p'}}{\partial x} - \frac{1}{\overline{\rho}}\frac{\partial \overline{v'p'}}{\partial y} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z}}_{(VI)} \underbrace{- \epsilon}_{(VII)} \; .
\end{aligned} \tag{13}
$$

The meaning of each term is given here: (I) time rate of change of TKE, (II) advection of TKE by the mean wind, (III) buoyancy generation / destruction, (IV) shear production of TKE, (V) transport of TKE by turbulent motions, (VI) transport of TKE by pressure, (VII) dissipation of TKE

Note here that terms (II), (V), and (VI) neither create or destroy TKE, they simply move it within the boundary layer. There is no Coriolis term, its effects cannot generate TKE. Term (III) describes TKE generation/destruction by buoyancy, while term (IV) describes TKE production by velocity gradients. Dissipation must generally be parameterized, and represents the transfer of energy to the molecular scale. Thus, it is acts as a TKE destruction term.

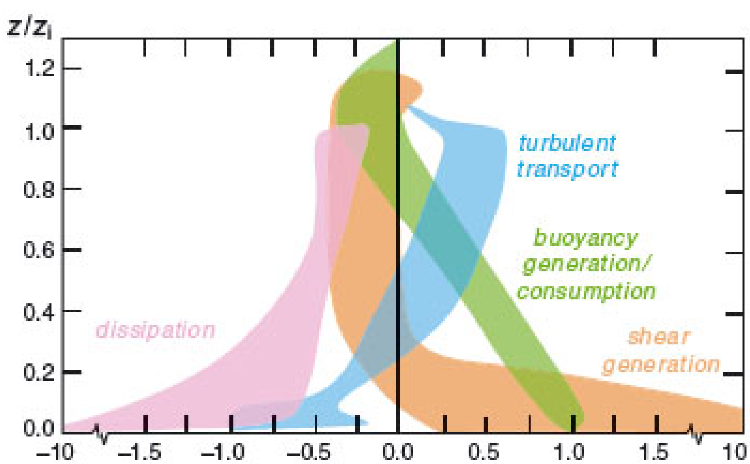

Figure 4: Typical ranges of terms in the TKE budget equation during daytime, composited from observations and numerical simulations from a number of investigators. [From Markowski and Richardson]

3.3.4 Surface Energy Budget

The exchange of heat and moisture between the surface and overlying atmosphere is responsible for the diurnal variations in boundary layer temperature, humidity, and depth. As we covered earlier, heat fluxes from the surface play a major role in generating boundary layer turbulence. Thus, it is worthwhile to examine the energy balance at the surface. It is through this so-called *surface energy budget* that surface heat and moisture fluxes are inescapably tied to the net radiation received at the surface.

The surface mainly receives predominantly short-wave radiation from the sun, with the amount absorbed by the ground dependent on the cloud fraction, solar angle, and surface albedo. The ground also receives predominantly long-wave (infrared) radiation emitted by clouds and the atmosphere. However, the earth's surface also emits radiation at long wavelengths. The net radiation, R_n, is the difference between the incoming short-wave and long-wave radiation and the outgoing long-wave radiation.

The sensible heat flux, Q_h, is related to the heating of the atmosphere from below. Air is largely transparent to incoming solar (short-wave) radiation and therefore is not heated directly by solar radiation; rather, diurnal boundary layer warming occurs via the heat flux convergence term ($-\partial\overline{w'\theta'}/\partial z$).

The latent heat flux, Q_e, on the other hand, represents the portion of the net radiation used in evaporation, transpiration, or the melting of ice at the surface. There also is a small, non-negligible downward flux of heat into the ground. This ground heat flux (Q_g) is usually relatively small compared with the sensible heat flux.

The surface energy budget, that is, the relationship between the net radiation and the sensible, latent, and ground heat fluxes, can be expressed as

$$R_n = Q_h + Q_e + Q_g \, , \tag{14}$$

where R_n is defined to be positive when incoming radiation exceeds outgoing radiation, and the heat fluxes are defined to be positive when directed *away* from the surface (*i.e.*, Q_h and Q_e are positive when directed upward, and Q_g is positive when directed downward).

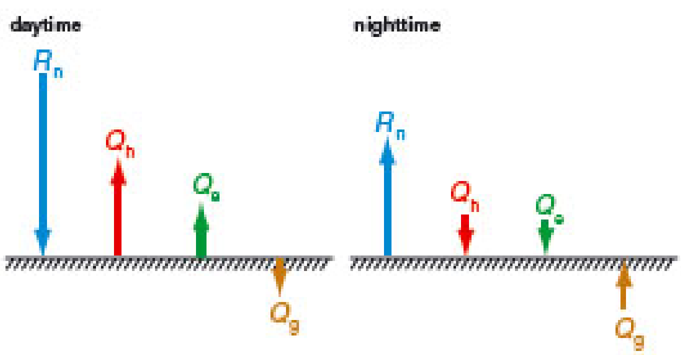

Figure 5: Schematic representation of the surface energy budget during daytime and nighttime. Actual magnitudes of the terms depend on the type of surface and its characteristics (e.g., soil type, soil moisture, vegetation), time of year, time of day, and weather. [From Markowski and Richardson]

Q_g is generally the smallest of the three surface fluxes. Its exact value depends on the soil type and moisture content (10% of the R_n).

Q_e depends on the amount of available surface moisture, which is a function of vegetation, soil wetness, land use, and near-surface wind speed. It can range from ~ 0 in a desert to more than 400 W/m^2 in a jungle.

Q_h depends on the temperature difference between the surface and the air, as well as the wind speed. Daytime values can range from 10 to more than 500 W/m^2.

The ratio of sensible and latent heat fluxes is known as the Bowen ratio

$$B = \frac{Q_h}{Q_e}. \tag{15}$$

The larger the Bowen ratio, the larger the amount of sensible heating of the lower atmosphere for a given net radiation and ground heat flux.

Bowen ratios are typically smallest over oceans, where $B \sim 0.1$.

On the other hand, $B \sim 0.5$ over forests and $B \sim$ 3-5 in arid regions.

In deserts or regions of severe drought, $B > 10$ has been observed!

The Bowen ratio can have large local diurnal variations, depending on precipitation. For example B drops after heavy rains.