LES of Turbulent Flows: Lecture 19

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Overview

1. Statistical Conditions for a SGS Model
2. Computing SGS Quantities

Statistical Conditions for a SGS Model

a posteriori

- Project #1 is based on *a posteriori* model testing – one particular way to test for a model's correctness
- The idea there is to use the model in a simulation and compare the output data with experimental data
- A downside to this testing approach is that it is hard to understand what physics are important for a model to work (or to fail)
- Another downside is the difficulty in separating the model from other influences, such as model numerics

Statistical Conditions for a SGS Model

a priori

- Project #2 will be based on *a priori* model testing – a way to test for a model's physical robustness
- The idea there is to use some fully-resolved "truth" dataset (DNS, experiments) and filter them to some chosen scale
- The filtered data is used to compute the actual and modeled stresses, which are then compared

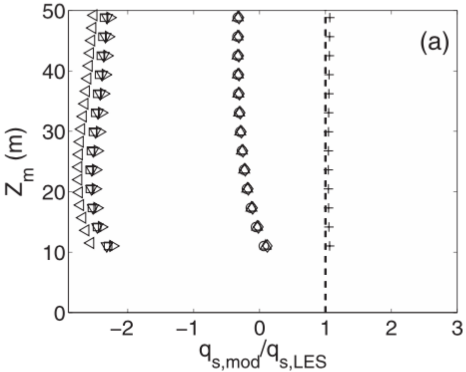
Statistical Conditions for a SGS Model

a priori

- A downside to *a priori* model testing is that instantaneous comparison between real and modeled stresses often show poor correlation
- However, this does not necessarily translate into a "bad" result when performing *a posteriori* tests
- In other words, *a priori* tests can present an overly negative outlook for a model
- Thus, the use of both testing procedures is often required to make a fully-formed assessment of a model

Statistical Conditions for a SGS Model

What do we mean? See this from Stoll and Porté-Agel (2009)



Offline testing of schemes led to wrong sign of surface flux! But when coupled in a system, it can produce reasonable results.

Statistical Conditions for a SGS Model

- Can we get around these issues related to *a priori* tests?
- Yes! We can apply weaker conditions on our model that are ususal expected for *a priori* tests
- The truth is that for many applications we simply do not care about a single realization of an LES
- Why? Well, we expect the flow to evolve chaotically – so to expect perfect instantaneous relationships is not realistic
- Rather, we really care about whether the LES produces correct statistical features of a flow

Statistical Conditions for a SGS Model

- Switching to a statistical *a priori* framework leads to an obvious question - **what conditions should a SGS model satisfy?**
- Specifically we are interested in answering the question what statistical properties should τ_{ij} and τ_{ij}^M share?
- In other words, what specific properties must the model satisfy such that the real and LES low-order velocity statistics match in a reasonable way?

Statistical Conditions for a SGS Model

We know a "good" model should adhere to our equations of motion

- Invariance to translation, rotation, and reflection (in the absence of boundaries)
- Hopefully, invariance to Re
- Ideally, invariant to Δ

To get more specific than this, we need to talk about **statistics of SGS models** (Meneveau 1994)

We want to know what properties τ_{ij} and τ_{ij}^M should share to produce reasonably accurate ensemble statistics of the velocity field

Statistical Conditions for a SGS Model

- **To obtain correct 1$^{\text{st}}$- and 2$^{\text{nd}}$-order moments** of our resolved field, our **model must** at least be able to **produce average modeled stresses** that match the "real" stresses
- This **doesn't guarantee that our 2$^{\text{nd}}$-order moments are correct** – it is only a necessary condition everywhere

Statistical Conditions for a SGS Model

- **To produce 2^{nd}-order moments**, we need to have our model reproduce 2^{nd}- and 3^{rd}-order SGS stats including stresses and correlations (*e.g.* stress with velocity or stress with rate-of-strain tensor) that match the "real" values– This includes **matching $\langle \boldsymbol{\Pi} \rangle$ everywhere** (recall that Π is the SFS dissipation rate)
- For even higher order moments we need to match higher order SGS stats

Computing SGS Quantities

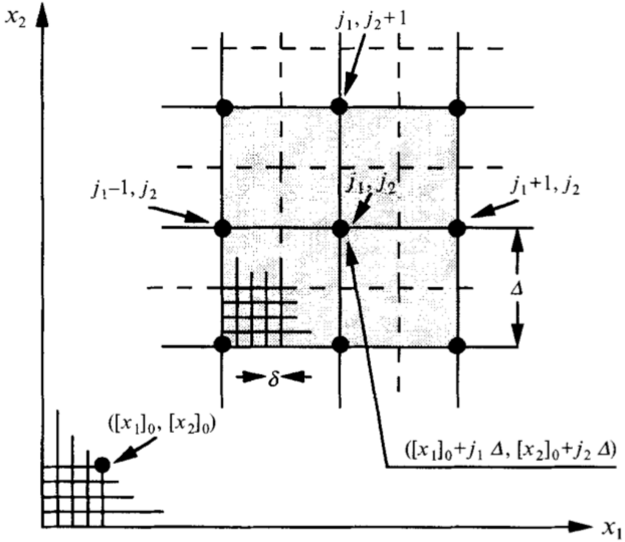
- Procedurally, **how do we compute these SGS stats from data** (DNS or experiments)?
- Select your data (after quality control) and identify missing velocity or gradient terms
- Separate the data into resolved and SGS scales by calculating $\tilde{u}_i$ and $\widetilde{u_i u_j}$ with an appropriate LES filter (see lecture 5 for the most common examples)

Computing SGS Quantities

- At this point, a decision must be made: **to down-sample or not** (see Liu et al. 1994)
- Down-sampling means removing points from the field that are separated (spatially) by less than our filter scale Δ (denoted by the $\sim$)
- Effectively this means we keep less points than we started with (e.g. from 128^3 to 32^3) after filtering

Computing SGS Quantities

Figure 6 from Liu et al. (1994)



Computing SGS Quantities

Downsampling

- **Pros**: we get a "true" representation of the effect of gradient estimates on our SGS models and avoid enhanced correlations due to filter overlap
- **Cons**: we lose data points (important if we have limited data) and we now need to consider the above gradient estimation errors!

Computing SGS Quantities

- Calculate **local values** of all the components of

$$\tau_{ij}^{\Delta} = \widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j$$

  and

$$S_{ij} = \left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i} \right)$$

- You may need approximations here based on your data!
- For some models you may need to calculate other parameters (*e.g.*, mixed and nonlinear models) but the general procedure is the same

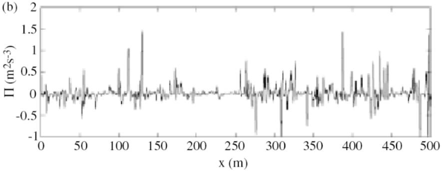
Computing SGS Quantities

- Homework #2 implemented different types of filters
- Once you have these basic quantities calculated you can calculate model values of $\tau_{ij}^{\Delta,M}$ and statistics of the actual (from data) and modeled SGS stresses including average values, correlation coefficients, and variances
- We can also calculate other SGS statistics like $\langle \Pi^{\Delta} \rangle = -\langle \tau_{ij}^{\Delta} \widetilde{S}_{ij} \rangle$ and $\langle \Pi^{\Delta,M} \rangle$ – or any model coefficients of interest
- The following pages give some examples of SGS statistics and model coefficients calculated from various references

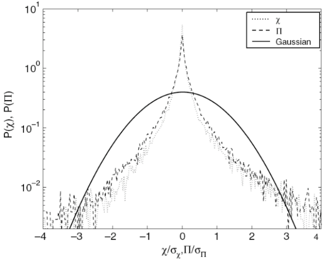
SGS Dissipation $\Pi = -\tau_{ij}\widetilde{S}_{ij}$

Π from experiments in the Utah desert (Carper & Porté-Agel 2004)

Experimental setup



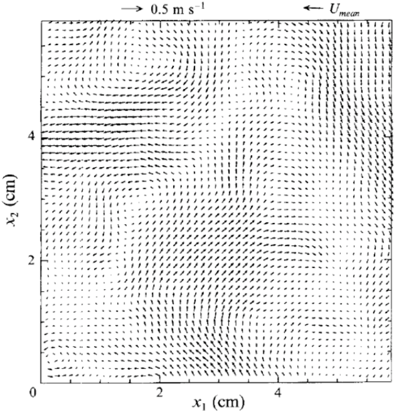
Example time series of Π from the ABL (late afternoon)



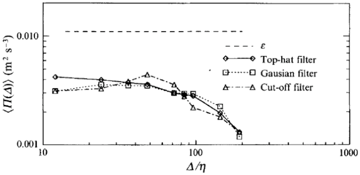
Example PDF of Π from the ABL (late afternoon)

SGS Dissipation $\Pi = -\tau_{ij}\widetilde{S}_{ij}$

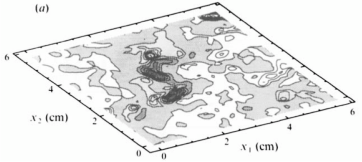
Π from wind tunnel experiments in a round jet (Liu et al. 1994)



Top-hat filtered PIV field



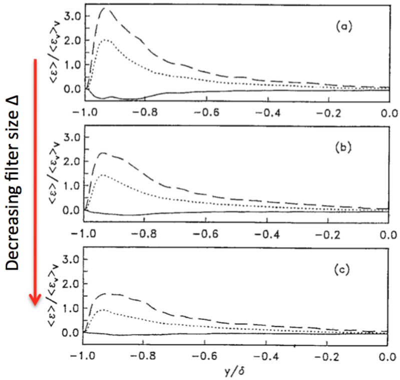
Average Π from the wind tunnel experiment compared to molecular dissipation



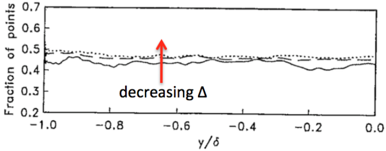
Spatial distribution of Π from PIV

SGS Dissipation $\Pi = -\tau_{ij}\widetilde{S}_{ij}$

Π DNS of turbulent channel flow Re=3300 (U_c) (Piomelli et al. 1991)



Π normalized by the total dissipation
— average; — — rms and · · · backscatter



Fraction of points in channel flow with backscatter for 3 different filter widths

- backscatter increases with Re
- fraction of backscatter points decrease for a Gaussian filter (cutoff results shown) to about 30%.

SGS SGS Model Correlation Coefficients

Correlation coefficients from Clark et al (1979) for different models

Evaluation of subgrid-scale models 13

Term	Model	Correlation $\frac{17}{8}\Delta$	Correlation $\frac{9}{8}\Delta$	Model constant $\frac{17}{8}\Delta$	Model constant $\frac{9}{8}\Delta$
τ_{ij} (tensor)	Smagorinsky	0·366	0·277	0·270	0·247
	Vorticity	0·344	0·260	0·294	0·275
	Turbulent kinetic energy	0·363	0·303	0·196	0·175
	Eddy viscosity	0·352	0·295		
$\frac{\partial \tau_{ij}}{\partial x_j}$ (vector)	Smagorinsky	0·425	0·346	0·240	0·264
	Vorticity	0·408	0·327	0·220	0·247
	Turbulent kinetic energy	0·434	0·362	0·138	0·155
	Eddy viscosity	0·426	0·356		
$u_i \frac{\partial \tau_{ij}}{\partial x_j}$ (scalar)	Smagorinsky	0·710	0·580	0·186	0·171
	Vorticity	0·700	0·582	0·202	0·191
	Turbulent kinetic energy	0·723	0·606	0·085	0·095
	Eddy viscosity	0·716	0·605		

TABLE 2. Summary of correlations between exact subgrid-scale Reynolds stresses and models.

- Eddy-viscosity - $\tau_{ij} = -2\nu_T \widetilde{S}_{ij}$
- Smagorinsky - $\nu_T = (C_s \Delta)^2 |\widetilde{S}_{ij}|$
- Deardorff - $\nu_T = (C_1 \Delta)^2 \widetilde{k}_r^{1/2}$
- Vorticity - $\nu_T = (C\Delta)^2 (\omega_i \omega_i)^{1/2}$

SGS SGS Model Correlation Coefficients

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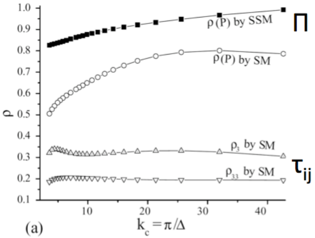
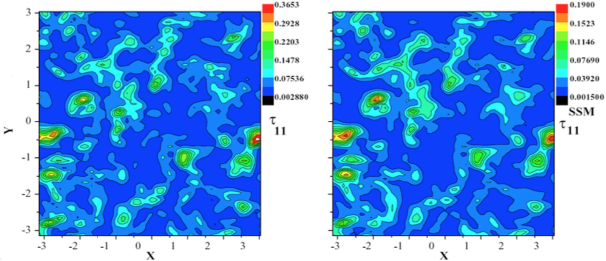


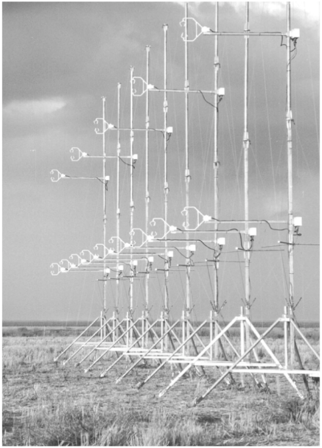
Correlation coefficients from Lu et al (2007) for Smagorinsky and Similarity models



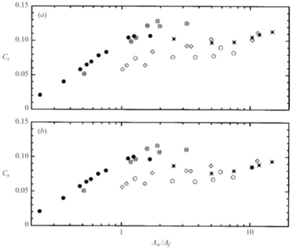
Measured (left) and modeled (right) with the similarity model τ_{11} from Lu et al (2007)

SGS SGS Model Coefficient Estimation

Model coefficients evaluated by matching Π from ABL study of Sullivan et al (2003)

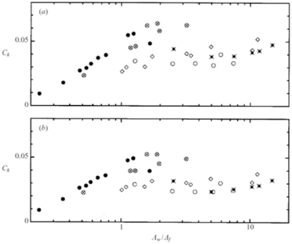


Experimental setup in Colorado



Smagorinsky

Mixed model



Kinetic Energy

Mixed KE

Smagorinsky coefficients with stability (Kleissl et al 2004)

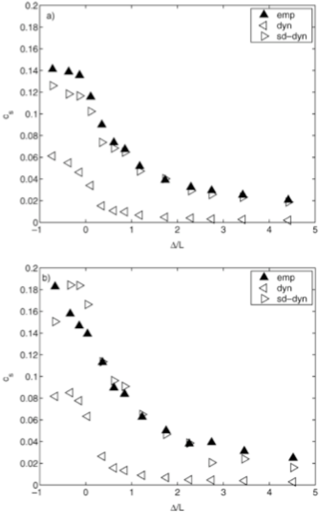
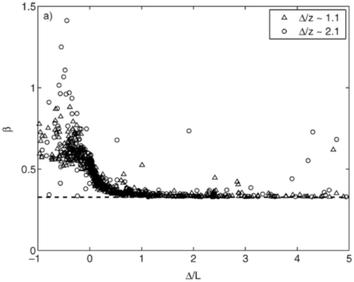


FIG. 14. Smagorinsky coefficient $c_s^{(\Delta)}$ as a function of Δ/L for different SGS models. Variables are averaged over all segments in each stability bin. (a) Array 1, $\Delta/z \sim 2.1$ and (b) array 2, $\Delta/z \sim 1.1$.



$$\frac{C_S^2(2\Delta)}{C_S^2(\Delta)} = \beta$$

SGS SGS Model Coefficient Estimation

Smagorinsky coefficients with stability (Bou-Zeid et al. 2010)

$q_i^{\text{model}} = -k_{\text{SGS}} \partial \tilde{\theta} / \partial x_i = -\text{Pr}_{\text{SGS}}^{-1} (C_S \Delta)^2 |\tilde{S}| \partial \tilde{\theta} / \partial x_i$

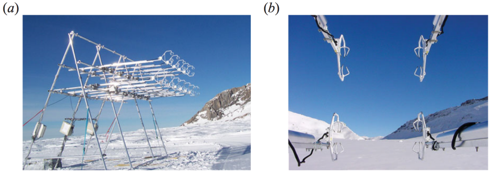


FIGURE 1. SnoHATS: side view of the 12 sonics array (*a*) and the upwind fetch of 1.5 km (*b*).

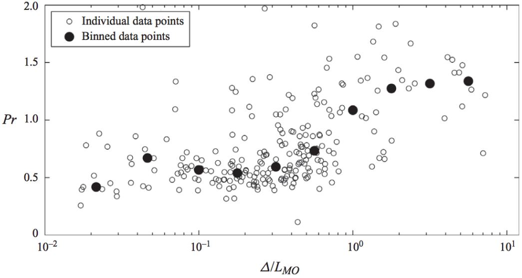


FIGURE 13. Variation of the SGS Prandtl number with the stability parameter based on the Obukhov scale.

SGS SGS Model Coefficient Estimation

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$q_i^{\text{model}} = -k_{\text{SGS}} \partial\tilde{\theta}/\partial x_i = -\text{Pr}_{\text{SGS}}^{-1} (C_S\Delta)^2 |\tilde{S}| \partial\tilde{\theta}/\partial x_i$

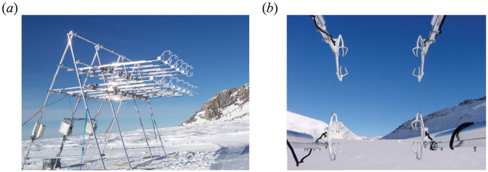


FIGURE 1. SnoHATS: side view of the 12 sonics array (*a*) and the upwind fetch of 1.5 km (*b*).

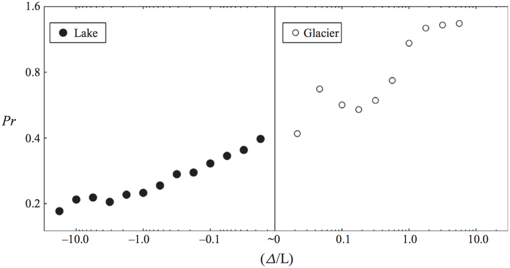
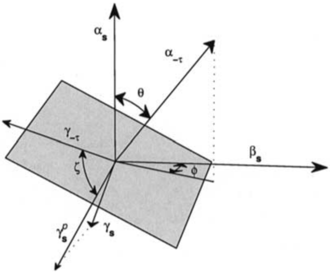


FIGURE 20. Variation of the SGS Prandtl number for unstable and stable conditions (note that both axes are in logarithmic scale).

Geometric Tensor Alignment

Higgins et al (2003)



Definition of the 3 angles needed to characterize the alignment of 2 tensors (τ_{ij} and S_{ij}

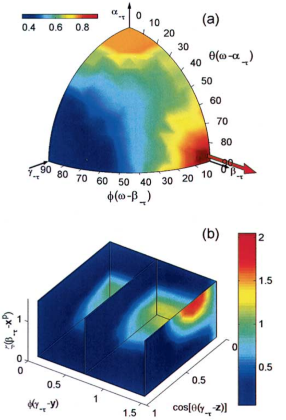


Figure 9. (a) Joint probability density function of two angles describing the orientation of filtered fluctuating vorticity vector in local negative SGS stress-tensor eigensystem. Filter scale is $\Delta = 2$ m. The filtered fluctuating vorticity has a bimodal distribution giving two likely alignment configurations. The primary alignment is between filtered fluctuating vorticity and the intermediate eigendirection of the SGS stress, $\boldsymbol{\beta}_{-\tau}$. The secondary alignment is vorticity and the extensive eigendirection of the negative SGS stress, $\boldsymbol{\alpha}_{-\tau}$. (b) Joint PDF of three angles describing the orientation of SGS stress tensor eigensystem in mean flow frame of reference. Filter scale is $\Delta = 2$ m. The mean flow streamwise direction is x, z is the vertical axis, and y is the transverse horizontal direction. The planes represent slices through the three-dimensional function.

Geometric Tensor Alignment

SGS and coherent structures in the Utah desert (Carper and Porté-Agel 2004)

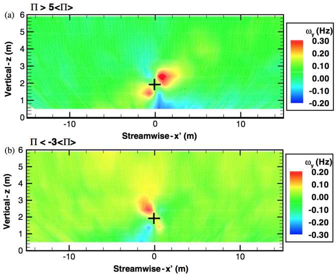


Figure 10. Vertical fields of conditionally averaged spanwise vorticity ω_y (Hz) for positive and negative SFS dissipation rates of energy under weakly unstable conditions (data-set D1). The '+' indicates the location at which the SFS dissipation rates satisfy the threshold condition.

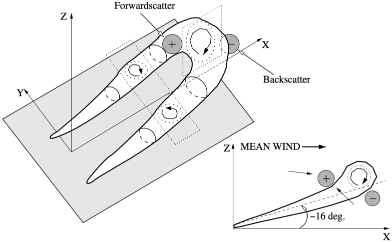


Figure 14. Conceptual model relating strong positive (+) and negative (−) SFS dissipation events to different regions (shaded) around a hairpin-like coherent structure. The solid lines outline an isosurface of vorticity with arrows indicating the direction of rotation. The dotted lines indicate the planes on which the conditionally averaged fields are reported with the key results shown within dashed circles.