LES of Turbulent Flows: Lecture 11

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Fall 2016

Overview

1. LES and numerical methods
2. Subgrid-Scale Modeling
3. Modeling τ_{ij}

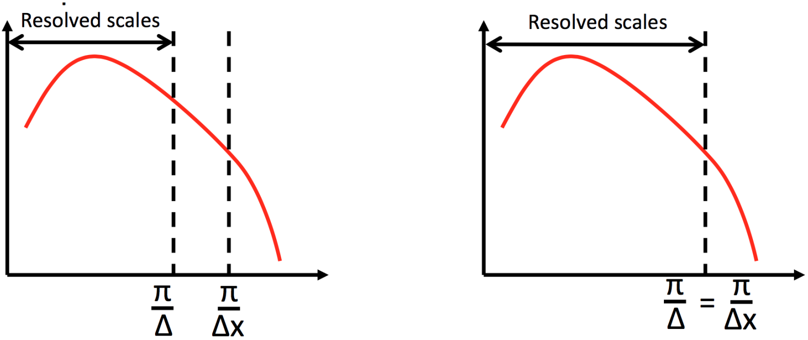
LES and numerical methods

- LES requires that the filtered equations of motion (see Lectures 6 and 7) be solved on a numerical grid
- In LES we need to accurately represent high wavenumber turbulent fluctuations (small scale turbulence)
- This means either we use high-order schemes (*e.g.*, spectral methods) or we use fine grids with low-order schemes (*e.g.*, 2^{nd}-order central differences)

LES and numerical methods

- High-order schemes are more expensive, but for a given mesh they are more accurate (see Pope 571-579 for a discussion of resolving filtered fields)
- Low-order finite difference (or volume) schemes provide flexibility of geometry but give rise to complications when modeling small scales motions
- For more complete reviews of LES and numerics, see Guerts chapter 5 and Sagaut chapter 8.2, 8.3

LES and numerical methods



LES and numerical methods

- The filter applied in LES can be either **implicit** or **explicit**
- **Implicit filtering:** The grid (or numerical basis) is assumed to be the LES low-pass filter
- **Explicit filtering:** A filter (typically box or Gaussian) is applied to the numerical grid (*i.e.*, explicitly to the discretized N-S equations)

LES and numerical methods

Implicit filtering

- **Pros:** takes full advantage of the numerical grid resolution
- **Cons:** for some methods it is helpful to know the shape of the LES filter (this can be difficult to determine for some numerical methods). Truncation error can also become an issue.

LES and numerical methods

Explicit filtering

- **Pros:** truncation error is reduced, filter shape is well defined
- **Cons:** loss of resolution – the total simulation time goes up as Δ_g^4 (where Δ_g is the grid spacing) so maintaining the same space resolution as an implicit filter with $\Delta_g/\Delta = 0.5$ will take $2^4 = 16$ more grid points

LES and numerical methods

- **Truncation errors can be on the order of SGS contributions** for low-order finite-difference schemes unless the filter width Δ is considerably larger than the grid spacing (see Ghosal 1996, Chow and Moin 2003)
- Ghosal and Chow suggested a filter-to-grid ratio of 4 when using a 2^{nd}-order centered scheme.

Ghosal (1996) key findings

- Consider a von Kármán spectrum, *e.g.*,

$$E(k) = \frac{ak^4}{(b+k^2)^{17/6}}$$

  The discretization error exceeds the subgrid error for 2nd-8th-order schemes. Additionally, for spectral schemes, the aliasing error (if 3/2 rule isn't used) will dominate subgrid errors.

- Reducing the ratio $C = \Delta_g/\Delta$ to values less than 1 reduces the error faster (by a factor of $C^{-3/4}$ than increasing the order of accuracy (a factor of 2 reduction when moving from 2nd to 8th)

LES and numerical methods

- 2^{nd}-order schemes may have undesirable truncation errors (with respect to SFS model terms)
- However, even-order schemes are non-dissipative
- SFS models (on average) are purely dissipative
- Therefore, <u>all hope is not lost</u>!

LES and numerical methods

- The same is not true for odd-order schemes common in compressible flow solutions.
- Examples include upwind-biased, total variation diminishing (TVD), or fixed least squares (FLS) schemes
- See *Numerical Methods for Conservation Laws* by Leveque (1992) for a review of this numerical approach
- Using these types of schemes with LES is somewhat controversial since their dissipative nature introduces an eddy viscosity like term to the solution (more later)

- The total numerical dissipation in many cases introduced by upwind schemes is greater than that of SFS viscosity models (if no pre-filtering is performed) – even for 7th-order schemes (Beaudan and Moin, 1994)
- For the Smagorinsky model (more later) Garnier et al. (1999) found that up to 5th-order upwind schemes in decaying isotropic turbulence are more dissipative
- For the Deardorff scheme, Gibbs and Fedorovich (2014b) found that the same result when applied to an atmospheric convective boundary layer (CBL)

LES and numerical methods

Gibbs and Fedorovich (2014b) overview

- Studied two idealized convective boundary layers – one with a mean shear of $10\ \mathrm{m\ s^{-1}}$ and the other with no mean shear
- Each case was run with our traditional LES code (OU-LES) and the WRF model used in LES mode (WRF-LES) on identical $10.24 \times 10.24 \times 2\ \mathrm{km}^2$ numerical grids with 20-m spacing
- OU-LES used 2$^{\text{nd}}$-order centered finite difference approximations, WRF-LES used 5$^{\text{th}}$-order upwind-biased finite differences
- Compared turbulence spectra and other statistics

LES and numerical methods

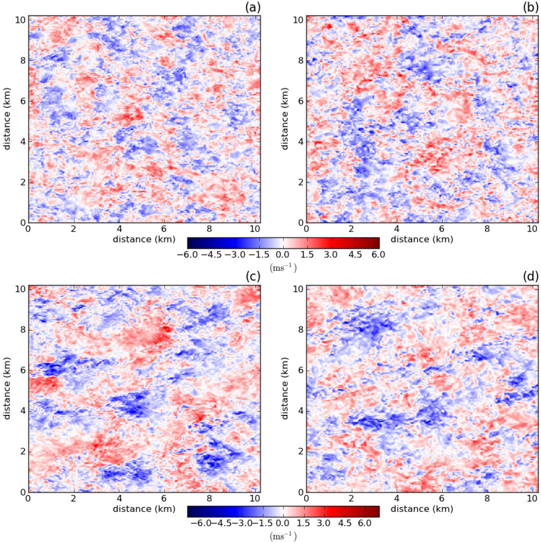


Figure: *u*-component velocity for shear-free (top) and shear-driven (bottom) from OU-LES (left) and WRF-LES (right) at $z/z_i = 0.25$

LES and numerical methods

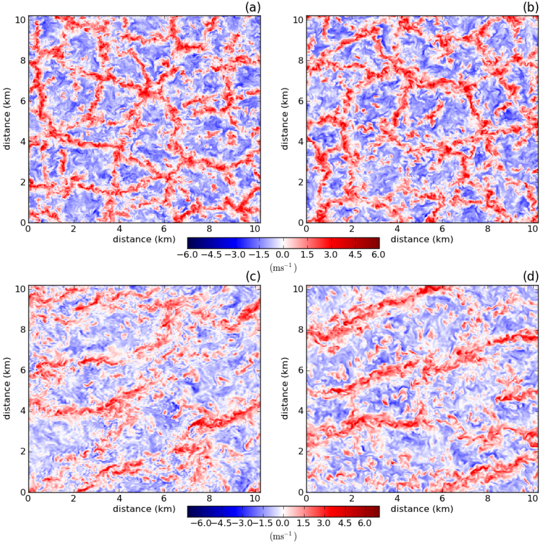


Figure: *w*-component velocity for shear-free (top) and shear-driven (bottom) from OU-LES (left) and WRF-LES (right) at $z/z_i = 0.25$

LES and numerical methods

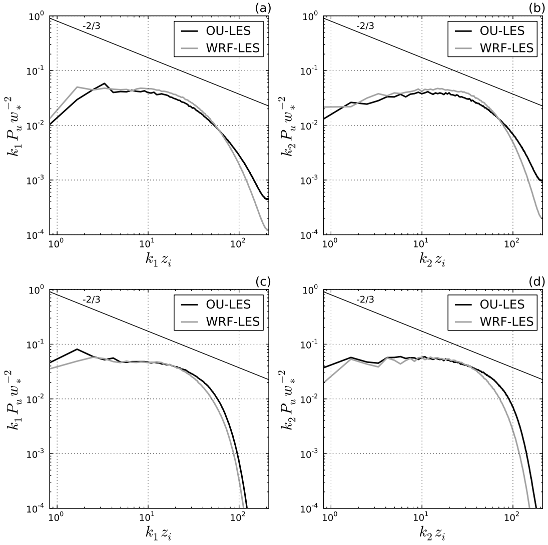
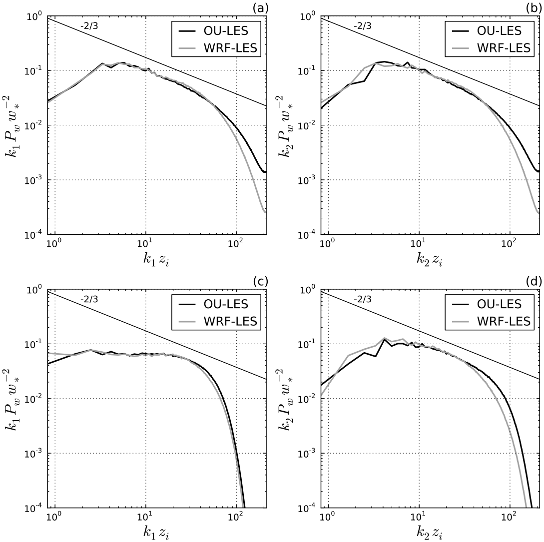


Figure: normalized 1D u-component spectra (left: x-dir, right: y-dir) for shear-free (top) and shear-driven (bottom) from OU-LES (left) and WRF-LES (right) at

LES and numerical methods



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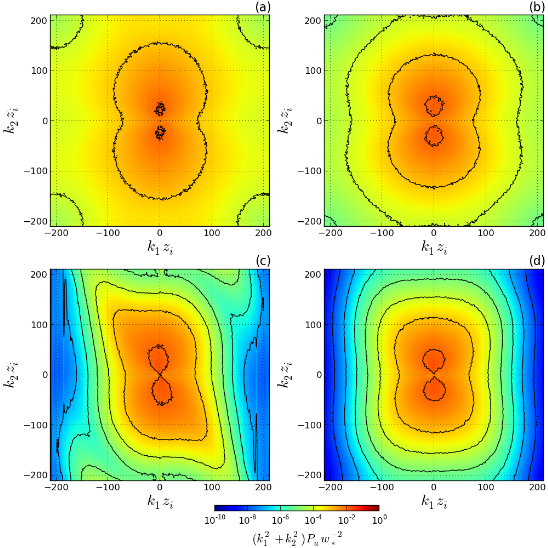
LES and numerical methods



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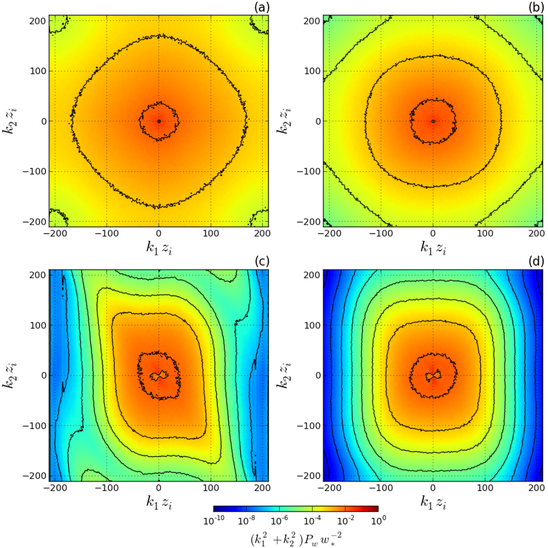
LES and numerical methods



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LES and numerical methods

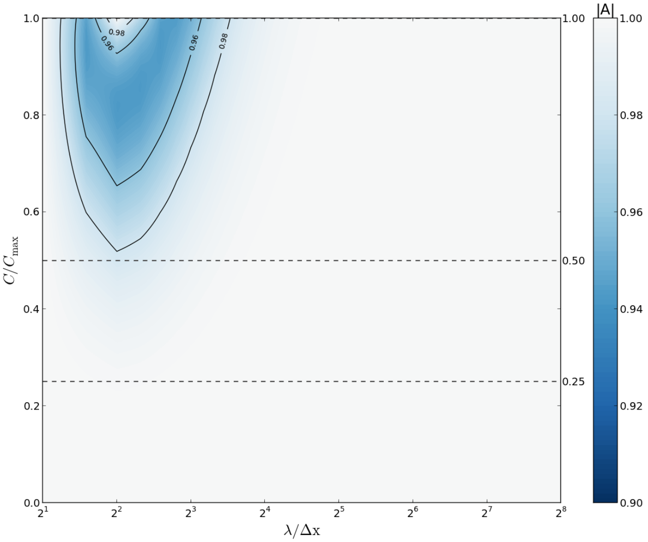


Figure: Amplification factor for 2nd-order centered scheme with RK3 time scheme.

LES and numerical methods

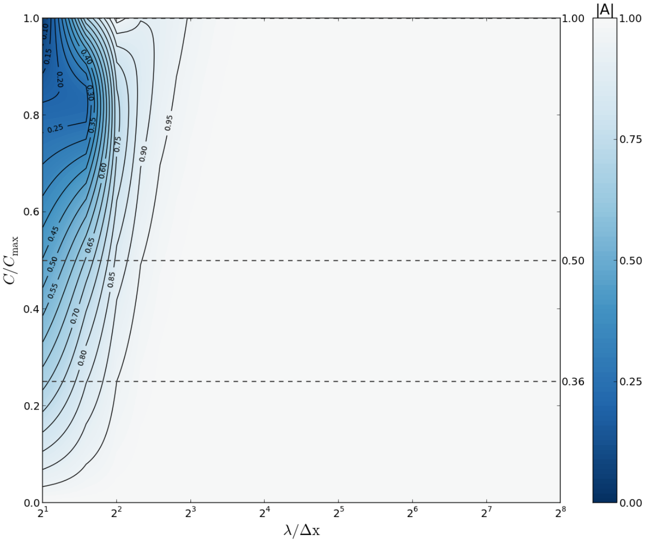


Figure: Amplification factor for 5^{th}-order upwind scheme with RK3 time scheme.

LES and numerical methods

- Gibbs and Fedorovich (2014a) looked at the effect of time integration schemes on turbulence spectra and moments
- You can see how many things complicate the understanding of LES results – the model itself, the choice of cutoff scale, the model numerics, etc.
- These items can also act to determine the spectral resolution of the simulation – meaning what minimum scale of features is trustworthy from a theoretical and statistical standpoint

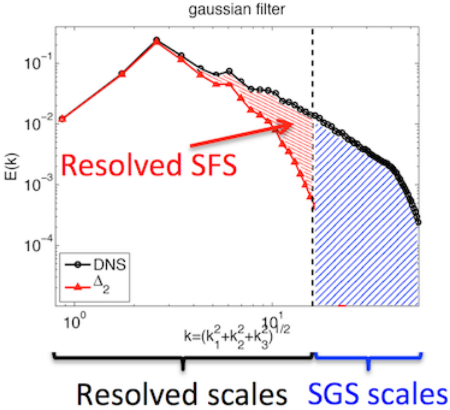
Subgrid-Scale Modeling

- One of the major hurdles to making LES a reliable tool for engineering and environmental applications is the formulation of SGS models and the specification of model coefficients

Subgrid-Scale Modeling

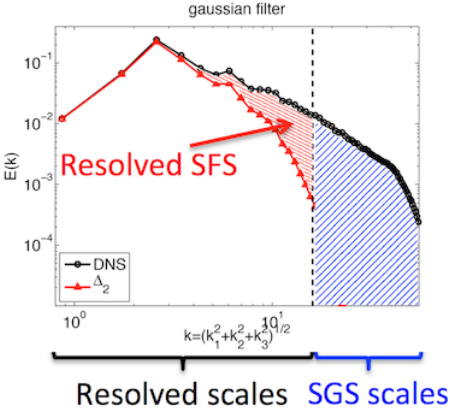
- Recall: we can define 3 different "scale regions" in LES
  - resolved scales
  - resolved SFS
  - SGSs
- We can also decompose a general variable as

$$\phi = \widetilde{\phi} + \phi'$$



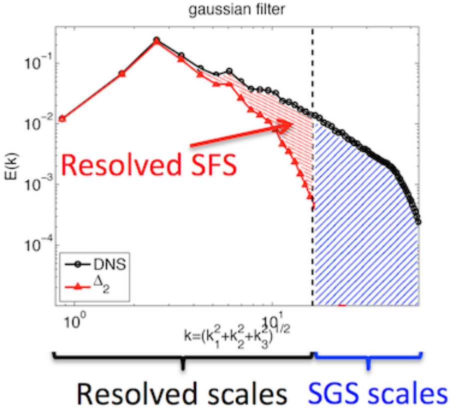
Subgrid-Scale Modeling

- When we talk about SGS models we are specifically talking about the scales below Δ and **NOT**the resolved SFS
- We will specifically discuss the resolved SFSs when we talk about filter reconstruction later on (time permitting)



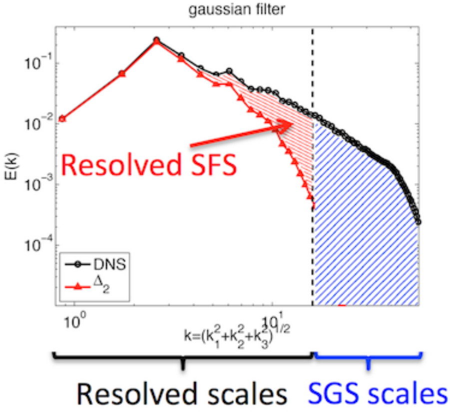
Subgrid-Scale Modeling

- We will focus on LES with explicit SGS models
- A class of LES referred to as Implicit LES (ILES) also exists
- Note: these terms are different than implicit/explicit filtering



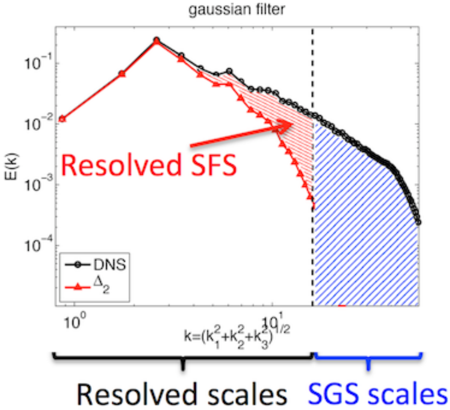
Subgrid-Scale Modeling

- ILES was first developed for compressible flow
- ILES methods are numerical methods that capture the energy-containing and inertial ranges of turbulent flows, while relying on their own intrinsic dissipation to act as a subgrid model



Subgrid-Scale Modeling

- ILES assumes the SGSs are purely dissipative and act in a similar way to dissipative numerical schemes
- In general, ILES uses monotinicity-preserving numerical schemes
- See Grinstein *et al.* (2007) on Canvas and website



Modeling τ_{ij}

- Our discussion of modeling τ_{ij} will follow Sagaut (pages 49-50, 59-60) and Pope (pages 582-583)
- We can decompose the nonlinear term as follows by using $\phi = \widetilde{\phi} + \phi'$

$$
\begin{aligned}
\widetilde{u_i u_j} &= \widetilde{\left(\widetilde{u}_i + u_i'\right)\left(\widetilde{u}_j + u_j'\right)} \\
&= \widetilde{\widetilde{u}_i \widetilde{u}_j} + \widetilde{\widetilde{u}_i u_j'} + \widetilde{\widetilde{u}_j u_i'} + \widetilde{u_i' u_j'}
\end{aligned}
$$

Modeling τ_{ij}

$$
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\end{aligned}
$$

- We now have the nonlinear term as a function of $\widetilde{u}_i$ and $\widetilde{u_i'}$
- Two different basic forms of the decomposition (based on the above equation) are prevalent

Modeling τ_{ij}

- The first one is based on the idea that all terms appearing in the evolution of a filtered quantity should be filtered

$$\tau_{ij} = \boxed{C_{ij} + R_{ij}} = \widetilde{u_i u_j} - \widetilde{\widetilde{u}_i \widetilde{u}_j}$$

where $C_{ij} = \widetilde{\widetilde{u}_i u'_j} + \widetilde{\widetilde{u}_j u'_i}$ $\Rightarrow$ interaction between resolved and SFSs

and $R_{ij} = \widetilde{u'_i u'_j}$ $\Rightarrow$ SFS "Reynold's" stress

Modeling τ_{ij}

- A second definition can be obtained by further decomposition of $\widetilde{\widetilde{u}_i\widetilde{u}_j}$

$$\widetilde{\widetilde{u}_i\widetilde{u}_j} = \underbrace{\left(\widetilde{\widetilde{u}_i\widetilde{u}_j} - \widetilde{u}_i\widetilde{u}_j\right)}_{L_{ij}} + \widetilde{u}_i\widetilde{u}_j$$

where $L_{ij} \Rightarrow$ Leonard stress – the interaction among the smallest resolved scales)

Our total decomposition is now

$$\boxed{\tau_{ij} = L_{ij} + C_{ij} + R_{ij}} = \widetilde{u_i u_j} - \widetilde{u}_i\widetilde{u}_j$$

- See Leonard (1974)

Modeling τ_{ij}

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij}$$

- If our filter is a Reynold's operator, the C_{ij} and L_{ij} vanish
- **Note:** while τ_{ij} and R_{ij} are invariant to Galilean transformations, L_{ij} and C_{ij} are not (see Speziale 1985)
- As a result, the decomposition given above (for the most part) is not used anymore
- However, we will see similar terms again in our SGS models

Modeling τ_{ij}

- Germano purposed a more rigorous decomposition into generalized filtered moments
- Under this framework, the generalized moments look just like "Reynolds" moments, our original τ_{ij}
- See Germano (1992) on Canvas and website
- This is also recommended reading for a discussion of filtering and the relationship between LES filters and Reynolds operators