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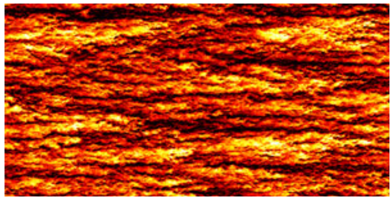
Challenging the large eddy simulation technique with advanced *a posteriori* tests



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The large eddy simulation (LES) technique will soon be 50 years old. Since Deardorff's first papers in 1970 introducing this approach, major advances in the theory of LES and its computational implementation have been made and widely adopted. However, in terms of validation, LES studies continue to largely focus on the first- and second-order statistics, which in fact are the same tests that Deardorff conducted 45 years ago. Further advances in LES and wider adoption for new flows require advanced and more challenging tests to be developed and documented to serve as benchmarks. The paper by Stevens, Wilczek & Meneveau (*J. Fluid Mech.*, 2014, vol. 757, pp. 888–907) does precisely that. The authors demonstrate the ability of LES to capture the recently established log-law of streamwise velocity variance and the related log-laws for even-order statistics up to order 10, as well as the departure of these statistics from a Gaussian distribution. The paper also provides key insights into the role of grid resolution on the computed turbulence field.

Key words: turbulence simulation, turbulent boundary layers

1. Introduction

The first large eddy simulation (LES) of a three-dimensional turbulent flow was reported in the very early 1970s in the pioneering paper of Deardorff (1970), based on prior foundational work by Joseph Smagorinsky and Douglas Lilly. Deardorff was then at the National Centre for Atmospheric Research; the problem he was most interested in is the very high-Reynolds-number flow ($Re \sim 10^8$) in the planetary boundary layer (PBL). At that time, PBL turbulence and how it is influenced by rotation and buoyancy were poorly understood. Therefore, appealing features of LES, compared to the more established Reynolds-averaged Navier Stokes (RANS) approach (see the early review by Reynolds 1976), would have been (i) the potentially reduced role of the turbulence closure model and (ii) the ability to obtain some estimates of higher-order turbulence statistics. Another feature of interest that Deardorff explicitly mentions is (iii) the feasibility of simulating very high-Re flows (compared to direct numerical simulations (DNS), which had already been carried out). These three features have since contributed to making LES an appealing simulation technique for a large set

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of turbulent flows in many fields. The surge of LES studies is intimately linked to the substantial development in the theory underlying the technique (e.g. subgrid-scale (SGS) modelling and the role of explicit or implicit filters), as well as the increasing ability to conduct the numerical solution of the filtered equations accurately. Despite these advances some questions on the limits and best practices of the technique remain open (see the 'ten questions' of Pope 2004). Addressing these questions will be increasingly important if LES is to become a more robust technique that can be applied to new problems more confidently and with reduced validation constraints.

Towards that goal, various studies have conducted *a priori* and *a posteriori* testing of LES. *A priori* tests use theory, experimental data or DNS to establish a set of conditions that should be met by an LES to reproduce the physics of the flow correctly. These tests, however, have limitations (see Meneveau 1994), and it cannot be guaranteed that their 'recommendations' necessarily result in improved performance in an actual simulation. The ultimate test of an LES must therefore consist of comparing actual simulation results to measurements, DNS or theory; this is known as *a posteriori* testing. These tests have historically focused on the first- and second-order statistics due to experimental errors that particularly affect higher-order statistics and the limitation of available theoretical results, like the mean log-law or Kolmogorov's 5/3rd law, to these low-order statistics. With DNS, one is limited by the low *Re* and the increased sensitivity of high-order statistics to *Re*. This restricted range of available generalizable tests has reduced our ability to improve LES and to confidently set limits on its skills (What statistics can it capture? When are the resolved fractions sufficient? etc.). Against that background, the paper by Stevens, Wilczek & Meneveau (2014) makes a significant contribution by providing a valuable benchmark test for LES of wall-bounded flows, and uses this test to make important inferences about the role of grid resolution and the implied inner scale of the simulation.

2. Overview

Stevens *et al.* (2014) exploit the recently discovered log-law for the variance of the streamwise velocity in wall-bounded neutral (no buoyancy) flows:

$$\frac{\overline{u'^2}}{u_*^2} = B_1 - A_1 \log\left(\frac{y}{\delta}\right), \tag{2.1}$$

where the prime denotes the perturbation relative to the mean velocity; u_* is the friction velocity, y the vertical distance from the wall and δ the depth of the boundary layer (see experimental results in Marusic & Kunkel 2003, and theoretical foundation in Perry & Chong 1982 and Hultmark 2012). Here $A_1 \approx 1.25$ is a universal constant for wall-bounded flows, while B_1 varies with flow conditions. If the velocity statistics are assumed to be Gaussian, one can also relate the higher even-order statistics to those of the second order and derive log-laws for them in which the universal coefficients A_p, equivalent to A_1, can then be theoretically related to A_1.

Using wind tunnel data, Meneveau & Marusic (2013) had established that although log-laws are indeed observed for even-order statistics up to order 10, the measured coefficients indicate departure from Gaussianity, as expected. These findings provide the motivation for Stevens *et al.* (2014) to attempt to reproduce them with LES. As the authors point out, a well-established paradigm in LES is that the first-order means are largely resolved and not very sensitive to the SGS contribution and thus a code should aim to reproduce the energy cascade from resolved to subgrid scales to accurately capture the second-order turbulent energy (see Meneveau & Katz 2000). However, no conditions are explicitly imposed to ensure that statistics of order three or higher are correctly reproduced, and indeed there are studies that document that LES could match first- and second-order statistics accurately while simultaneously failing to capture higher orders (Pan, Chamecki & Isard 2014). Stevens *et al.* apply a well-tested LES code with advanced numerical schemes and dynamic SGS modelling (Bou-Zeid, Meneveau & Parlange 2005). They perform 20 simulations with increasing grid resolutions, up to a massive $2048 \times 1024 \times 577$ grid points. The two main parameters they vary are the roughness length z_0 and the grid scale Δ; these are the most obvious candidates to serve as an inner scale in wall-modelled LES where the fluid viscosity does not appear in the equations and no inner viscous scale exists. Here Δ also controls the fraction of the resolved turbulence.

The authors demonstrate that it is critical to resolve well over 90 % of the variance (their highest resolutions resolve over 98 %) for the LES to adequately reproduce the log-law for higher even-order moments. Since the resolved fraction decreases as the wall is approached, low resolutions were not able to reproduce a log-region or erroneously overestimated the height of its lower limit. This lower limit is then shown to depend on Δ but not on z_0, indicating that Δ is the effective inner scale of turbulence in these simulations. The central and major contribution of their paper is linked to the success of the LES in capturing remarkably well the log-regions for the streamwise velocity variance and even-order statistics up to order 10, as measured experimentally. Even more interesting perhaps is the fact that the universal coefficients A_p deduced from LES also match the wind tunnel data. This is far from being a trivial success since it requires the LES to accurately capture the departure of the velocity statistics from a Gaussian distribution.

3. Future

One can at present almost take for granted that in many flows that have been adequately investigated using LES, the mean and root-mean-square velocities produced by proper simulations will be in good agreement with theory and measurements (compressible or reacting flows remain more challenging). However, various studies have already documented some limitations of LES results, for example when considering structure functions and space–time correlations (Kang, Chester & Meneveau 2003; Yang, He & Wang 2008). Matching some lower-order statistics is thus no guarantee that all other statistics are also accurately simulated. The study of Stevens *et al.* therefore makes a noteworthy contribution by establishing a robust test case that can allow us to push LES outside its ‘comfort zone’. An appealing feature of this test case is that the results can be expressed as universal laws. Further similar generalized benchmarks are critically needed.

Another important lesson from Stevens *et al.* is that even higher resolutions are still needed. The author of this article collected data on the typical resolutions used in LES by searching the *Journal of Fluid Mechanics* for papers with ‘large eddy simulation’ in the title and randomly selecting data from papers published after 1980 (excluding papers that purposely use low resolution to test the limits of LES). Considering the reduction in time step needed when the total number of grid points N increases in LES, along with the rise in available computing power predicted (and realized) by Moore’s law, one would expect N to scale $\sim 2^{Y/2}$ in LES, where Y is the year. However, figure 1 suggests that LES grid resolutions have been increasing more slowly, roughly $\sim 2^{Y/3}$. The densest grid in Stevens *et al.* lies closer to Moore’s law and demonstrates the feasibility of such high resolutions. Tests from another high-resolution paper (Sullivan & Patton 2011), considering up to third-order moments, also confirm the importance of further higher resolutions in LES. This underlines the need to raise the bar not only for LES testing, but also for its grid resolutions.

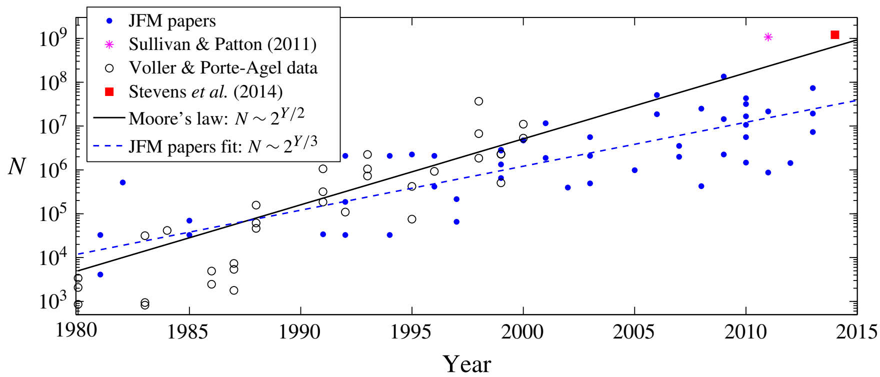

FIGURE 1. The increase in the number of grid nodes in LES in *J. Fluid Mech.* (list of article dois available as supplementary material at http://dx.doi.org/10.1017/jfm.2014.616), data from a study by Voller & Porte-Agel (2002) of computational fields that follow Moore's law more closely, and the grids of Stevens *et al.* and Sullivan & Patton (2011).

Supplementary material

Supplementary material is available at http://dx.doi.org/10.1017/jfm.2014.616.

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