Environmental Fluid Dynamics: Lecture 22

Dr. Jeremy A. Gibbs

Department of Mechanical Engineering
University of Utah

Spring 2017

Overview

Integral Forms of Flux-Profile Relationships

Integral Flux-Profile Relationships: Overview

- Dimensionless gradients of velocity, temperature, and moisture are universal functions of $\zeta = z/L$.
- We can vertically integrate these gradients to obtain explicit expressions for their profiles.
- When this is done, we will build in corrections for static stability.

Integral Flux-Profile Relationships: Momentum

- Integrate flux-profile equation from z_1 to $z_2 > z_1$ in the ASL

$$
\begin{aligned}
\frac{\kappa z}{u_*}\frac{\partial \overline{u}}{\partial z} &= \phi_m(\zeta) \\
\frac{\kappa z}{u_*}\frac{\partial \overline{u}}{\partial z} &= 1 - 1 + \phi_m(\zeta) \\
\frac{\partial \overline{u}}{\partial z} &= \frac{u_*}{\kappa}\left[\frac{1}{z} - \frac{1-\phi_m(\zeta)}{z}\right] \\
\int_{z_1}^{z_2} \frac{\partial \overline{u}}{\partial z} dz &= \frac{u_*}{\kappa}\left[\int_{z_1}^{z_2} \frac{1}{z} dz - \int_{z_1}^{z_2} \frac{1-\phi_m(\zeta)}{z} dz\right] \\
\overline{u}(z_2) - \overline{u}(z_1) &= \frac{u_*}{\kappa}\left[\ln\left(\frac{z_2}{z_1}\right) - \psi_m\left(\zeta_2, \zeta_1\right)\right]
\end{aligned}
$$

where

$$
\psi_m\left(\zeta_2, \zeta_1\right) = \int_{z_1}^{z_2} \frac{1-\phi_m(\zeta)}{z} dz
$$

Integral Flux-Profile Relationships: Momentum

$$\psi_m\left(\zeta_2,\zeta_1\right)=\int_{z_1}^{z_2}\frac{1-\phi_m(\zeta)}{z}dz$$

- Let's take $z_1 = z_0$ (where $\overline{u}=0$) and $z_2 = z > z_0$

$$\overline{u}(z)=\frac{u_*}{\kappa}\left[\ln\left(\frac{z}{z_0}\right)-\psi_m\left(\zeta,\zeta_0\right)\right]$$

- ψ_m is called the *stability correction function* and describes the deviation of the velocity profile from the log-law due to the effect of atmospheric stability.
- We generally take $\psi_m(\zeta_0)=0$ and $\psi_m=\psi_m(\zeta_0)$, which gives the common approximate form

$$\boxed{\overline{u}(z)=\frac{u_*}{\kappa}\left[\ln\left(\frac{z}{z_0}\right)-\psi_m(\zeta)\right]}$$

Integral Flux-Profile Relationships: Momentum

Stable

- Dyers function

$$\phi_m(\zeta) = 1 + 5\zeta \quad \text{where } \zeta \geq 0$$

- Stability correction function

$$\psi_m(\zeta) = \int_0^z \frac{1 - \phi_m(\zeta)}{z} dz$$
$$\psi_m(\zeta) = \int_0^z \frac{1 - (1 + 5z/L)}{z} dz$$
$$\psi_m(\zeta) = \int_0^z -\frac{5}{L} dz = -5\frac{z}{L}$$
$$\boxed{\psi_h(\zeta) = -5\zeta}$$

Integral Flux-Profile Relationships: Momentum

Unstable

- Dyers function

$$\phi_m(\zeta) = (1 - 16\zeta)^{-1/4} \quad \text{where } \zeta \leq 0$$

- Stability correction function

  Apply a similar procedure and use a standard integral table to arrive at

$$\boxed{\begin{array}{l} \psi_m(\zeta) = 2\ln\left(\dfrac{1+x}{2}\right) + \ln\left(\dfrac{1+x^2}{2}\right) - 2\tan^{-1}x + \dfrac{\pi}{2} \\ \\ \text{where } x = \phi_m^{-1} = (1 - 16\zeta)^{1/4} \end{array}}$$

Integral Flux-Profile Relationships: Momentum

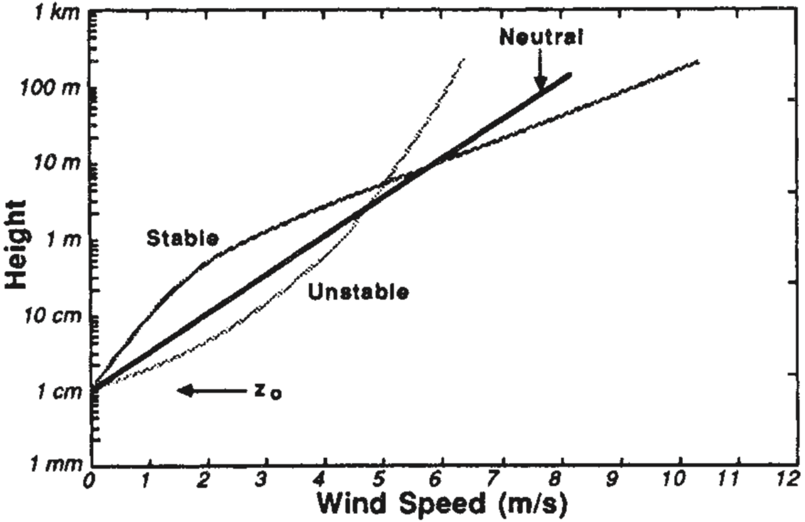
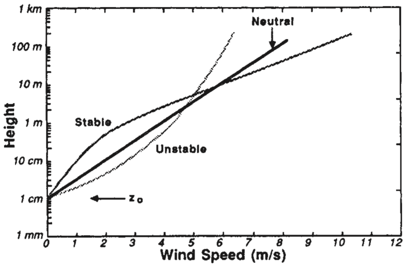


Fig 9.5 from Stull (1988)

Integral Flux-Profile Relationships: Momentum



- Deviations from the log-law increase with increasing $|\zeta|$
- Under stable conditions, the profiles are log-linear and tend to become linear for large ζ.
- For unstable conditions, $\psi_m, \psi_h > 0$, so deviations are negative. Accordingly, the profiles become increasingly curvilinear for large $|\zeta|$.

Integral Flux-Profile Relationships: Heat/Moisture

- Integrate flux-profile equation from z_1 to $z_2 > z_1$ in the ASL

$$
\begin{aligned}
\frac{\kappa z}{\theta_*}\frac{\partial\overline{\theta}}{\partial z} &= \phi_h(\zeta) \\
\frac{\kappa z}{\theta_*}\frac{\partial\overline{\theta}}{\partial z} &= 1 - 1 + \phi_h(\zeta) \\
\frac{\partial\overline{\theta}}{\partial z} &= \frac{\theta_*}{\kappa}\left[\frac{1}{z} - \frac{1-\phi_h(\zeta)}{z}\right] \\
\int_{z_1}^{z_2}\frac{\partial\overline{\theta}}{\partial z}dz &= \frac{\theta_*}{\kappa}\left[\int_{z_1}^{z_2}\frac{1}{z}dz - \int_{z_1}^{z_2}\frac{1-\phi_h(\zeta)}{z}dz\right] \\
\overline{\theta}(z_2) - \overline{\theta}(z_1) &= \frac{\theta_*}{\kappa}\left[\ln\left(\frac{z_2}{z_1}\right) - \psi_h\left(\zeta_2,\zeta_1\right)\right]
\end{aligned}
$$

where

$$
\psi_h\left(\zeta_2,\zeta_1\right) = \int_{z_1}^{z_2}\frac{1-\phi_h(\zeta)}{z}dz
$$

Integral Flux-Profile Relationships: Heat/Moisture

$$\psi_h\left(\zeta_2, \zeta_1\right) = \int_{z_1}^{z_2} \frac{1 - \phi_h(\zeta)}{z} dz$$

- Let's take $z_1 = z_{0\theta}$ (where $\overline{\theta} = \theta_s$) and $z_2 = z > z_{0\theta}$

$$\overline{\theta}(z) = \theta_s + \frac{\theta_*}{\kappa}\left[\ln\left(\frac{z}{z_{0\theta}}\right) - \psi_h\left(\zeta, \zeta_0\right)\right]$$

- Take $\psi_h(\zeta_{0\theta}) = 0$ and $\psi_h = \psi_h(\zeta_{0\theta})$, which gives the common approximate form

$$\boxed{\overline{\theta}(z) = \theta_s + \frac{\theta_*}{\kappa}\left[\ln\left(\frac{z}{z_{0\theta}}\right) - \psi_h(\zeta)\right]}$$

- Similarly,

$$\boxed{\overline{q}(z) = q_s + \frac{q_*}{\kappa}\left[\ln\left(\frac{z}{z_{0q}}\right) - \psi_h(\zeta)\right]}$$

Integral Flux-Profile Relationships: Heat/Moisture

Stable

- Dyers function

$$\phi_h(\zeta) = 1 + 5\zeta \quad \text{where } \zeta \geq 0$$

- Stability correction function

$$\psi_h(\zeta) = \int_0^z \frac{1 - \phi_h(\zeta)}{z} dz$$
$$\psi_h(\zeta) = \int_0^z \frac{1 - (1 + 5z/L)}{z} dz$$
$$\psi_h(\zeta) = \int_0^z -\frac{5}{L} dz = -5\frac{z}{L}$$
$$\boxed{\psi_m(\zeta) = -5\zeta}$$

Integral Flux-Profile Relationships: Heat/Moisture

Unstable

- Dyers function

$$\phi_h(\zeta) = (1 - 16\zeta)^{-1/2} \quad \text{where } \zeta \le 0$$

- Stability correction function

  Apply a similar procedure and use a standard integral table to arrive at

$$\boxed{\begin{array}{l} \psi_h(\zeta) = 2\ln\left(\dfrac{1+y}{2}\right) \\ \\ \text{where } y = \phi_h^{-1} = (1 - 16\zeta)^{1/2} \end{array}}$$

Calculation of Surface Turbulent Fluxes

- Here is a practical guide for how to compute surface turbulent fluxes in the case of non-coinciding measurements/model levels.
- In this scenario, we have mean values of u,T(abs. temp), and q measured at u_1, u_2 at z_{u1}, z_{u2}, $T_1 \approx \theta_1, T_2 \approx \theta_2$ at $z_{\theta 1}, z_{\theta 2}$, and q_1, q_2 at z_{q1}, z_{q2}.
- p is known at one of measurement level and β and ρ can also be evaluated.

Calculation of Surface Turbulent Fluxes

1. First approximation: u, θ, and q are neutral and logarithmic

$$u_2 - u_1 = \frac{u_*}{\kappa} \ln \frac{z_{u2}}{z_{u1}}$$
$$\theta_2 - \theta_1 = \frac{\theta_*}{\kappa} \ln \frac{z_{\theta 2}}{z_{\theta 1}}$$
$$q_2 - q_1 = \frac{q_*}{\kappa} \ln \frac{z_{q2}}{z_{q1}}$$

which gives a first guess of the turbulence scales

$$u_* = \frac{\kappa(u_2 - u_1)}{\ln(z_{u2}/z_{u1})}$$
$$\theta_* = \frac{\kappa(\theta_2 - \theta_1)}{\ln(z_{\theta 2}/z_{\theta 1})}$$
$$q_* = \frac{\kappa(q_2 - q_1)}{\ln(z_{q2}/z_{q1})}$$

Calculation of Surface Turbulent Fluxes

2. Based on those scales, evaluate the Obukhov length

$$L = \frac{u_*^2}{\kappa(\beta\theta_* + 0.61 g q_*)}$$

3. If $z_h/|L| \ll 1$ ($z_h =$ highest measurement level), the flow is considered neutral. It is reasonable to take $z_h/|L| = 0.01$ as the lower limit for the non-neutral case.

   In a near-neutral ASL ($z_h/|L| < 0.01$), kinematic fluxes are evaluated based on the computed scales.

$$\overline{w'u'} = -u_*^2$$
$$\overline{w'\theta'} = -u_*\theta_*$$
$$\overline{w'q'} = -u_* q*$$

Calculation of Surface Turbulent Fluxes

4. If $z_h/|L| \geq 0.01$, we proceed to new approximations of the turbulence scales

$$u_* = \frac{\kappa(u_2 - u_1)}{\left[\ln\left(\frac{z_{u2}}{z_{u1}}\right) - \psi_m(\zeta_{u2}) + \psi_m(\zeta_{u1})\right]}$$

$$\theta_* = \frac{\kappa(\theta_2 - \theta_1)}{\left[\ln\left(\frac{z_{\theta 2}}{z_{\theta 1}}\right) - \psi_h(\zeta_{\theta 2}) + \psi_h(\zeta_{\theta 1})\right]}$$

$$q_* = \frac{\kappa(q_2 - q_1)}{\left[\ln\left(\frac{z_{q2}}{z_{q1}}\right) - \psi_h(\zeta_{q2}) + \psi_h(\zeta_{q1})\right]}$$

making sure to account for the sign of L and applying the proper stability correction functions.

5. Based on the new scales, evaluate the Obukhov length

$$L = \frac{u_*^2}{\kappa(\beta\theta_* + 0.61gq_*)}$$

6. Steps 4 and 5 are repeated until the difference between the new and old values of L reach a minimum threshold (~ 0.01).
7. Based on computed scales, compute kinematic fluxes as

$$\overline{w'u'} = -u_*^2$$
$$\overline{w'\theta'} = -u_*\theta_*$$
$$\overline{w'q'} = -u_*q*$$

Calculation of Surface Turbulent Fluxes

8. Finally, u, θ, and q are obtained at any level z in the ASL:

$$u(z) = u_1 + \frac{u_*}{\kappa}\left[\ln\left(\frac{z}{z_{u1}}\right) - \psi_m(\zeta) + \psi_m(\zeta_{u1})\right]$$

$$\theta(z) = \theta_1 + \frac{\theta_*}{\kappa}\left[\ln\left(\frac{z}{z_{\theta 1}}\right) - \psi_h(\zeta) + \psi_h(\zeta_{\theta 1})\right]$$

$$q(z) = q_1 + \frac{q_*}{\kappa}\left[\ln\left(\frac{z}{z_{q1}}\right) - \psi_h(\zeta) + \psi_h(\zeta_{q1})\right]$$

Note: In this case, $z > z_{u1}, z_{\theta 1}, z_{q1}$. However, you can use values of u, θ, and q at some height $z_{u2}, z_{\theta 2}, z_{q2}$ to obtain their respective values at some height $z < z_{u2}, z_{\theta 2}, z_{q2}$