Environmental Fluid Dynamics: Lecture 19

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Overview

Stability Review

Stability Review

- In the broadest sense, stability is a dividing line between laminar and turbulent flows.
- If a flow is stable, it can become or remain laminar.
- If a flow is unstable, it can become or remain turbulent.
- There are many stabilizing and destabilizing factors to consider.

Stability Review

- These factors are often terms in the TKE balance equation
- To simplify and understand the problem, a destabilizing term is often paired with a stabilizing term to form a dimensionless ratio.
- This ratio is used to determine which factor "wins" and whether the flow becomes turbulent or not.
- Examples include the Reynolds number, Richardson number, Rossby number, Rayleigh number, and Froude number.

Stability Review

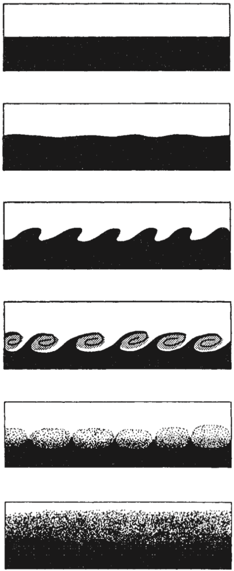
- Recall that we previously discussed static stability.
- "Static" means no motion, so it described stability that is independent of the wind.
- Static stability determined whether a flow was capable of buoyant convection.
- Hence, we framed our discussion of static stability around vertical profiles of potential temperature.
- Today we will discuss dynamic instability.

Dynamic Instability

Dynamic Instability

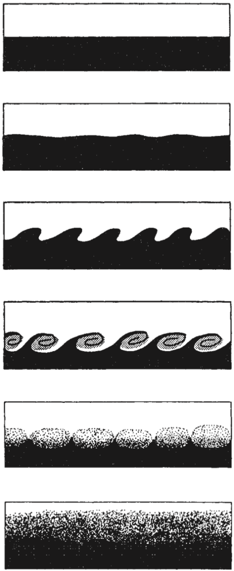
- "Dynamic" means having motion
- Unlike static stability, dynamic stability depends on the wind
- Even if the atmosphere is statically stable in some layer, wind shear may be sufficient to generate turbulence
- One example is Kelvin-Helmholtz instability

Dynamic Instability: Kelvin-Helmholtz



- Consider low density fluid on top of high density fluid (statically stable)
- Imagine that shear (ΔU) exists at their interface.
- If shear becomes large enough, the flow is dynamically unstable,
- The amplitude of the waves grows until they break.
- The breaking wave is called a Klevin-Helmholtz (KH) wave.
- The physics are different than a breaking wave on the ocean's surface, for instance.

Dynamic Instability: Kelvin-Helmholtz



- Within each wave roll there are "packets" of statically unstable fluid (locally heavy above light fluid)
- The static and dynamic instabilities act to generate turbulence.
- The turbulence expands and leads to diffusion (mixing), which transfers momentum and reduces the shear.
- If the shear falls below the critical value, then the dynamic instability ends.
- In the absence of the shear, turbulence decays and the flow becomes laminar.

Dynamic Instability: Kelvin-Helmholtz

- KH waves likely occur often in statically-stable shear layers, although they are rarely observed visually.
- If sufficient moisture is present in the atmosphere, clouds can form in the ascending portions of the wave.
- These are called billow clouds.

Dynamic Instability: Kelvin-Helmholtz

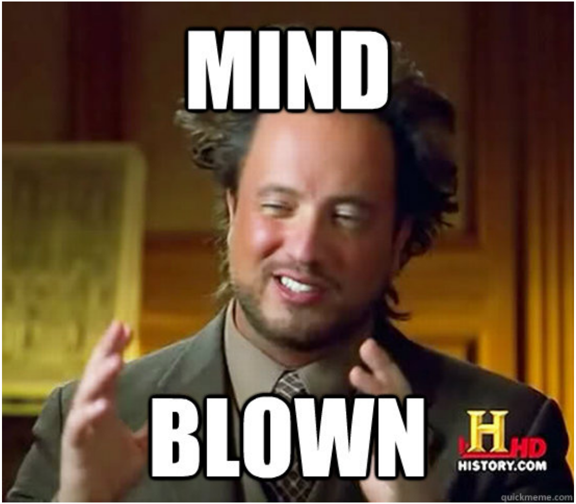
photo by Paul Chartier, via: Earthsky.org

Dynamic Instability

- Interestingly, both static and dynamic instabilities cause the flow to react in a way to remove the instability.
- So, turbulence is an "effort" by the flow to undo the cause of the instability.
- Static: convection moves buoyant air upward, which stabilizes the flow.
- Dynamic: turbulence reduces wind shears, which stabilizes the flow.

Dynamic Instability

- In other words, turbulence exists to end its own existance



Dynamic Instability

- Since observations show prolonged periods of turbulence in the atmosphere, there must be external forces that act to destabilize the PBL.
- Static: solar heating of ground.
- Dynamic: pressure gradients from synoptic-scale features causes winds to fight dissipation.

Dynamic Instability

- To understand when a flow might become dynamically unstable, we can compare the relative magnitudes of the shear and buoyancy terms in our TKE balance equation.
- Shear leads to mechanical production of turbulence - a destabilizing effect.
- Buoyancy can either enhance or suppress turbulence - so it can be stabilizing or destabilizing.
- One such ratio is the Richardson number Ri.

Richardson Number

Dynamic Instability: Flux Richardson Number

- Consider a statically-stable environment.
- turbulent motions have to fight gravity, so buoyancy suppresses turbulence here.
- Conversely, wind shear acts to generate turbulence.
- The buoyant production/consumption term in the TKE balance is negative and the shear production term is positive.
- It is useful to examine their ratio.

Dynamic Instability: Flux Richardson Number

- The ratio of buoyant production/consumption to shear production is called the **Flux Richardson Number**:

$$\mathrm{Ri_f} = \frac{\left(\dfrac{g}{\overline{\theta_v}}\right)(\overline{w'\theta_v'})}{(\overline{u_i'u_j'})\dfrac{\partial\overline{u_i}}{\partial x_j}} = \frac{(\overline{w'b'})}{(\overline{u_i'u_j'})\dfrac{\partial\overline{u_i}}{\partial x_j}}$$

- The Richardson number is dimensionless and contains nine terms int he denominator!

Dynamic Instability: Flux Richardson Number

- If we neglect subsidence ($\overline{w} = 0$) and assume horizontal homogeneity, we arrive:

$$\mathrm{Ri_f} = \frac{\left(\dfrac{g}{\overline{\theta_v}}\right)(\overline{w'\theta_v'})}{(\overline{u'w'})\dfrac{\partial \overline{u}}{\partial z} + (\overline{v'w'})\dfrac{\partial \overline{v}}{\partial z}} = \frac{(\overline{w'b'})}{(\overline{u'w'})\dfrac{\partial \overline{u}}{\partial z} + (\overline{v'w'})\dfrac{\partial \overline{v}}{\partial z}}$$

Dynamic Instability: Flux Richardson Number

- For statically stable, unstable, neutral flows, $\mathrm{Ri_f}$ is positive, negative, 0 - respectively.
- Richardson suggested $\mathrm{Ri_f} = 1$ is a critical value since mechanical and buoyant terms are in balance.
- At any value less than $\mathrm{Ri_f} = 1$, static stability is too weak to prevent mechanical generation of turbulence.
- For $\mathrm{Ri_f} < 1$, buoyancy contributes to the generation of turbulence.

$$\begin{cases} \text{Flow is dynamically unstable (turbulent)} & \mathrm{Ri_f} < 1 \\ \text{Flow is dynamically stable (laminar)} & \mathrm{Ri_f} > 1 \end{cases}$$

- Note: statically unstable flow is by definition dynamically unstable.

Dynamic Instability: Gradient Richardson Number

- With $\mathrm{Ri_f}$, notice that our terms involve turbulent fluxes.
- So, it determines when a turbulent flow becomes laminar, but not when a laminar flow becomes turbulent.
- Using the idea of K-theory, we can say that $-\overline{w'\theta_v'}$ is proportional to $\partial\overline{\theta_v}/\partial z$,$-\overline{u'w'}$ is proportional to $\partial\overline{u}/\partial z$, and $-\overline{v'w'}$ is proportional to $\partial\overline{v}/\partial z$.
- We can use these assumptions to create a new Richardson number.

Dynamic Instability: Gradient Richardson Number

- The **Gradient Richardson Number** is given by:

$$\mathrm{Ri} = \frac{\dfrac{g}{\overline{\theta_v}}\left(\dfrac{\partial\overline{\theta_v}}{\partial z}\right)}{\left(\dfrac{\partial\overline{u}}{\partial z}\right)^2 + \left(\dfrac{\partial\overline{v}}{\partial z}\right)^2}$$

- If you see Ri without any subscript - the authors probably mean the gradient Richardson number.

Dynamic Instability: Gradient Richardson Number

- Theoretical and experimental data suggest that laminar flow becomes turbulent when Ri is below a critical value Ri_c, while turbulent flow ceases at some termination value Ri_t.

$$\begin{cases} \text{Laminar flow becomes turbulent} & \mathrm{Ri} < \mathrm{Ri}_\mathrm{c} \\ \text{Turbulent flow becomes laminar} & \mathrm{Ri} > \mathrm{Ri}_\mathrm{t} \end{cases}$$

- There is still debate about what values should be used, but typically $\mathrm{Ri}_\mathrm{c} = 0.25$ and $\mathrm{Ri}_\mathrm{t} = 1$.
- There is a hysteresis since $\mathrm{Ri}_\mathrm{t} > \mathrm{Ri}_\mathrm{c}$

Dynamic Instability: Gradient Richardson Number

Why the hysteresis?

- The idea is that we need two conditions for turbulence: instability and a trigger mechanism.
- Consider KH waves as a trigger mechanism.
- In the absence of existing turbulence, Ri must fall well below $\mathrm{Ri_t}$ in order for KH waves to form.
- Experimental data suggests KH waves form when $\mathrm{Ri} < \mathrm{Ri_c}$.

Dynamic Instability: Gradient Richardson Number

Why the hysteresis?

- Thus, we have a hysteresis.
- In other words, the Richardson number of a non-turbulent flow must be lowered to Ri_c, while a turbulent flow can remain so until the Richardson number surpasses Ri_t.

Dynamic Instability: Bulk Richardson Number

- $\mathrm{Ri_c} \approxeq 0.25$ is theoretically based on local gradients of wind and temperature.
- In the real world, we don't really know those local gradients.
- However, we can approximate them using discrete height layers.
- For example, we can approximate $\partial\overline{\theta_v}/\partial z$ as $\Delta\overline{\theta_v}/\Delta z$.
- We can apply these approximations to the gradient Richardson number to create a new ratio.

Dynamic Instability: Bulk Richardson Number

- The **Bulk Richardson Number** is given by:

$$\mathrm{Ri_B} = \frac{\dfrac{g}{\overline{\theta_v}}\dfrac{\Delta\overline{\theta_v}}{\Delta z}}{\left(\dfrac{\Delta\overline{u}}{\Delta z}\right)^2 + \left(\dfrac{\Delta\overline{v}}{\Delta z}\right)^2} = \frac{g\Delta\overline{\theta_v}\Delta z}{\overline{\theta_v}\left[(\Delta\overline{u})^2 + (\Delta\overline{v})^2\right]}$$

- This is the form most often used by meteorologists.
- Observational and NWP data give wind and temperature at discrete points in the vertical.
- Note: $\Delta\overline{u} = \overline{u}(\text{top}) - \overline{u}(\text{bottom})$

Dynamic Instability: Bulk Richardson Number

- A few words of caution with $\mathrm{Ri_B}$ are warranted.
- The critical value is based on local gradients and not finite differences across layers.
- Gradients likely get "washed out" as the layer grows in thickness.
- Uncertainty arises in our ability to determine whether turbulence might form.
- Since some observations are spread far apart in the vertical, we must take care in interpreting the computed $\mathrm{Ri_B}$ when trying to characterize the considered flow.