Environmental Fluid Dynamics: Lecture 15

Dr. Jeremy A. Gibbs

Department of Mechanical Engineering
University of Utah

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Overview

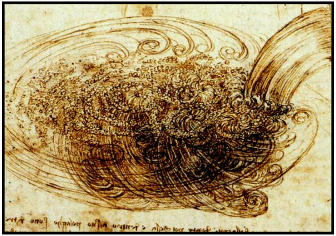
1. Intro to Turbulence
   - History
   - Motivation
   - Basic Properties of Turbulent Flows
2. Random Nature of Turbulence

Intro to Turbulence

Leonardo da Vinci and Turbulence

- Lived from 1452–1519.
- First to attempt scientific study of turbulence (*turbolenza*).
- He pioneered the notion of flow visualization to study turbulence.

Leonardo da Vinci and Turbulence



He let water flow through a square hole into a pool and observed:

> *Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion.*

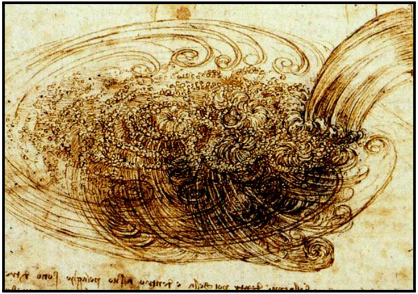
Leonardo da Vinci and Turbulence

In another, he placed obstacles in water:

> *So moving water strives to maintain the course pursuant to the power which occasions it, and if it finds an obstacle in its path it completes the span of the course it has commenced by a circular and revolving movement.*

Earliest reference to the importance of vortices!

Leonardo da Vinci and Turbulence



In another, he placed obstacles in water:

> *... the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones, and small things are turned by small eddies and large.*

Seems to hint at Richardson's turbulent cascade!

Why Turbulence?

Why study turbulence?

- Turbulence is everywhere.
- Smoke from a chimney, water flowing in a river, wind across a rough surface, flow around vehicles, combustion, solar wind.
- Most real flows in environmental and engineering applications are turbulent.
- It remains one of the great unsolved problems in physics.

Turbulence is Hard

Werner Heisenberg, maybe:

> *When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first.*

Horace Lamb, maybe:

> *I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.*

What Makes Turbulence Such a Difficult Problem?

Beyond these properties, consider turbulent flow in general.

- To understand turbulence, we must resolve the entire range of temporal and spatial scales of the flow.
- This range may be described by the Reynolds number (Re$=UL/\nu$) – the ratio of inertia to viscous forces.
- As Re increases, the range of length scales that must be increases dramatically.

What Makes Turbulence Such a Difficult Problem?

These scales often fall outside of those conditions that are easily measured:

- high-velocities (aerodynamics)
- high-temperatures (combustion)
- hazardous substances (nuclear engineering)
- very large scales (geophysics or astrophysics)

Approaches to Study Turbulence

Some common approaches to studying turbulence

- **Analytical**
  Due to the large range of scales and complexity of the flow, it is difficult (impossible really) to obtain an analytical solution
- **Computational**
  If we have large enough computing resources to resolve all scales of motion, we can solve the Navier-Stokes equations directly (DNS)
- **Empirical**
  Describing turbulence in a statistical framework

Approaches to Study Turbulence: Analytical

The Komogorov scales (more later) are defined as

- length scale

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$$

- time scale

$$\tau = \left(\frac{\nu}{\epsilon}\right)^{\frac{1}{2}}$$

- velocity scale

$$v = \frac{\eta}{\nu} = (\nu\epsilon)^{\frac{1}{4}}$$

We can relate these to the Reynolds number as

$$\eta \sim \ell_o \mathrm{Re}^{-\frac{3}{4}}$$

$$v \sim U_o \mathrm{Re}^{-\frac{1}{4}}$$

$$\tau \sim \frac{\ell_o}{U_o} \mathrm{Re}^{-\frac{1}{2}}$$

Approaches to Study Turbulence: Analytical

- Consider typical atmospheric scales:

$$U_o \sim 10 \text{ m s}^{-1},\ \ell_o \sim 10^3 \text{ m},\ \nu \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

- which gives us,

$$\mathsf{Re} = \frac{U_o \ell_o}{\nu} \sim \frac{(10 \text{ m s}^{-1})(10^3 \text{ m})}{10^{-5} \text{ m}^2 \text{ s}^{-1}} \sim 10^9$$

- thus,

$$\eta \sim \ell_o \mathsf{Re}^{-\frac{3}{4}} \sim 0.00018 \text{ m}$$
$$v \sim U_o \mathsf{Re}^{-\frac{1}{4}} \sim 0.06 \text{ m s}^{-1}$$
$$\tau \sim \frac{\ell_o}{U_o} \mathsf{Re}^{-\frac{1}{2}} \sim 0.003 \text{ s}$$

That is quite the range of scales!

Approaches to Study Turbulence: Analytical

- Beyond the large range of scales that must be described, think about the complexity of flows
- For an analytical solution, we would have to understand and describe all of those situations
- Okay, let's try brute force!

Approaches to Study Turbulence: Computational

- To capture all of the dynamics (degrees of freedom) in a turbulent flow, we must consider the required amount of discrete values needed for an accurate approximation.
- We need a grid fine enough to capture the smallest *and* the largest scales of motion (η and ℓ_o).

Approaches to Study Turbulence: Computational

- From K41, we know that $\ell_o/\eta \sim \text{Re}^{3/4}$ and there exists a continuous range of scales between η and ℓ_o.
- We will assume that we need n grid points per increment η. Note that n can vary, but a value of 3 to 5 is often suggested.
- Thus, in each direction, the number of required grid points is

$$N_i = \frac{\ell_o}{(\eta/n)} = n\,\frac{\ell_o}{\eta} \sim n\ \text{Re}^{3/4}$$

- Remember that turbulence is 3D, so the total number of grid points needed to accurately estimate the flow is

$$N = \left(n\ \text{Re}^{3/4}\right)^3 = \boxed{n^3\ \text{Re}^{9/4}}$$

Approaches to Study Turbulence: Computational

- Let's revisit our example of a typical atmospheric boundary layer flow:

$$U_o \sim 10 \text{ m s}^{-1},\ \ell_o \sim 10^3 \text{ m},\ \nu \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

which gives us,

$$\text{Re} = \frac{U_o \ell_o}{\nu} \sim \frac{(10 \text{ m s}^{-1})(10^3 \text{ m})}{10^{-5} \text{ m}^2 \text{ s}^{-1}} \sim 10^9$$

- thus, the number of grid points required to fully resolve this flow (assuming $n = 3$) is

$$N = 9 \times (10^9)^{9/4} \sim 1.6 \times 10^{21}!!!!!!!$$

Note: current capabilities of modern computing allow for grid sizes with $\mathcal{O}(10^{11})$ points.

Approaches to Study Turbulence: Computational

- What does a simulation of a typical atmospheric boundary layer flow using a grid with 1.6×10^{21} points buy? (recall $\eta \sim 0.18$ mm)

  $$l_i = \eta * \left(1.6 \times 10^{21}\right)^{1/3} \sim 2 \text{ km}$$

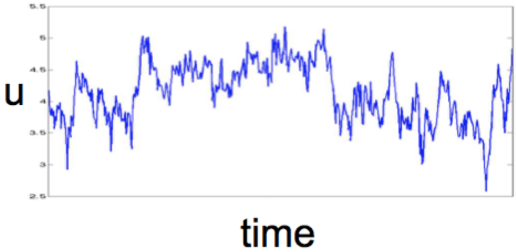
  This means we can simulate a $2 \text{ km} \times 2 \text{ km} \times 2 \text{ km}$ cube. Think how about big the atmosphere is and then be depressed.

Approaches to Study Turbulence: Statistical

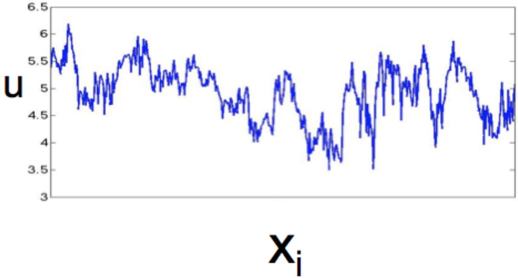
Okay, direct computational methods are out, that leaves the statistical approach

- "Separate" the flow into "mean" and "turbulent" parts (Reynolds decomposition, more later)
- Will require the use of averaging procedures (more later)

Basic Properties of Turbulent Flows



- Unsteady



- Three-dimensional

Basic Properties of Turbulence

- **Turbulence is random**
  The properties of the fluid (ρ, P, u) at any given point ($\vec{x}$,t) cannot be predicted. But statistical properties – time and space averages, correlation functions, and probability density functions – show regular behavior. The fluid motion is stochastic.
- **Turbulence decays without energy input**
  Turbulence must be driven or else it decays, returning the fluid to a laminar state.

Basic Properties of Turbulence

- **Turbulence displays scale-free behavior**
  On all length scales larger than the viscous dissipation scale but smaller than the scale on which the turbulence is being driven, the appearance of a fully developed turbulent flow is the same.
- **Turbulence displays intermittency**
  "Outlier" fluctuations occur more often than chance would predict.
- **Turbulence is non-linear**
  Growth of small perturbations, non-linear vortex stretching

Basic Properties of Turbulent Flows

- Large vorticity

- Vorticity describes the tendency of something to rotate.

$$
\begin{aligned}
\omega &= \nabla \times \vec{u} \\
&= \epsilon_{ijk} \frac{\partial}{\partial x_i} u_j \hat{e}_k \\
&= \left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \right) \hat{e}_1 + \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) \hat{e}_2 + \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x2} \right) \hat{e}_3
\end{aligned}
$$

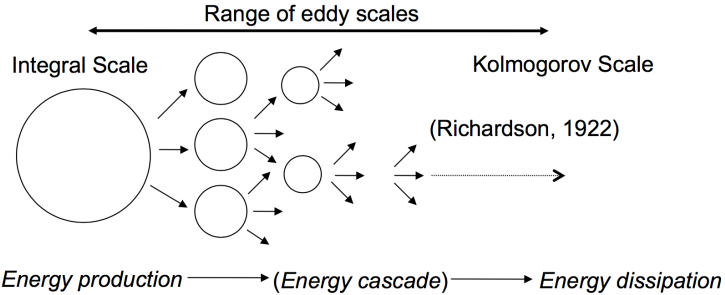
Vortex stretching can and does create small scale circulations that increases the turbulence intensity I, where:

$$I = \frac{\sigma_u}{\langle u \rangle}$$

Basic Properties of Turbulent Flows

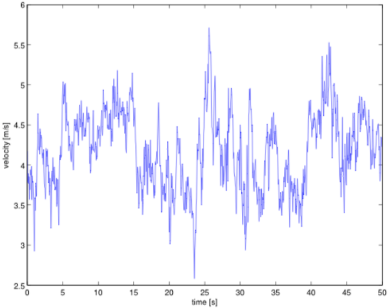
- Mixing effect

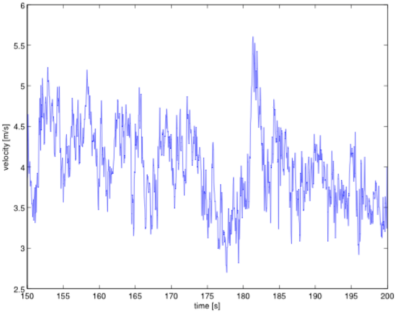
  Turbulence mixes quantities (*e.g.*, pollutants, chemicals, velocity components, etc)., which acts reduce gradients. This lowers the concentration of harmful scalars, but increases drag.
- A continuous spectrum (range) of scales.



Random Nature of Turbulence

Random Nature of Turbulence

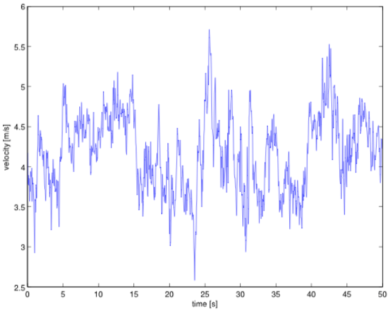


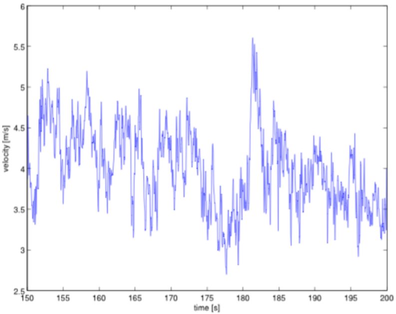


Sonic anemometer data at 20Hz taken in the ABL.

This velocity field exemplifies the random nature of turbulent flows.

Random Nature of Turbulence

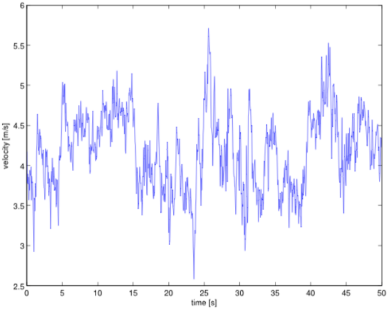


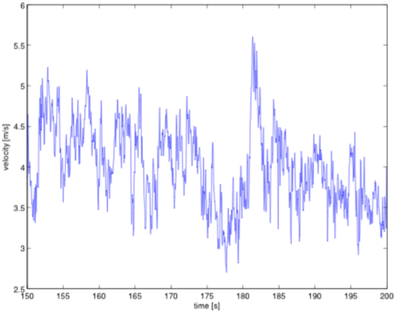


- The signal is highly disorganized and has structure on a wide range of scales (that is also disorganized).

  Notice the small (fast) changes verse the longer timescale changes that appear in no certain order.

Random Nature of Turbulence

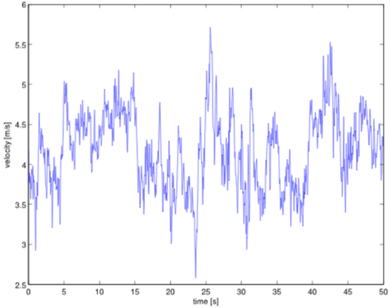


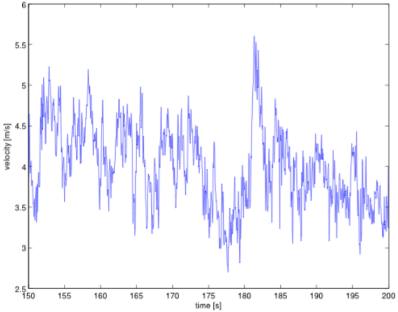


- The signal appears unpredictable.

  Compare the left plot with that on the right (100 s later). Basic aspects are the same but the details are completely different. From looking at the left signal, it is impossible to predict the right signal.

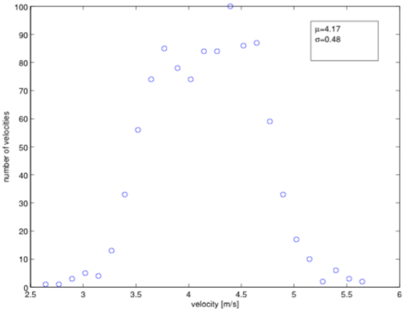
Random Nature of Turbulence

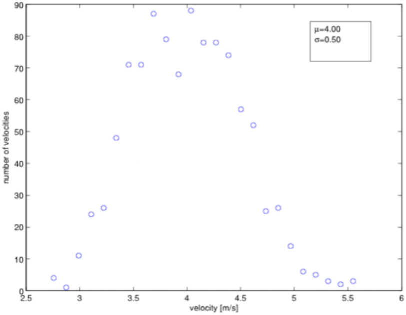




- Some of the properties of the signal appear to be reproducible.

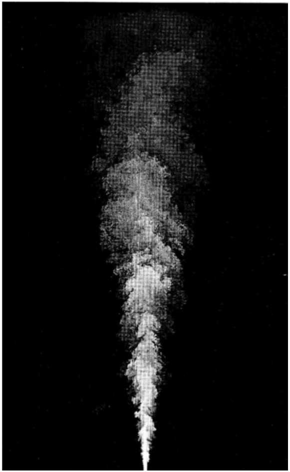
  The reproducible property isn't as obvious from the signal. Instead we need to look at the histogram.

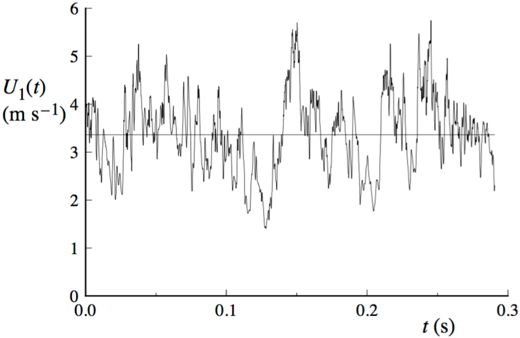




Notice that the histograms are similar with similar means and standard deviations.

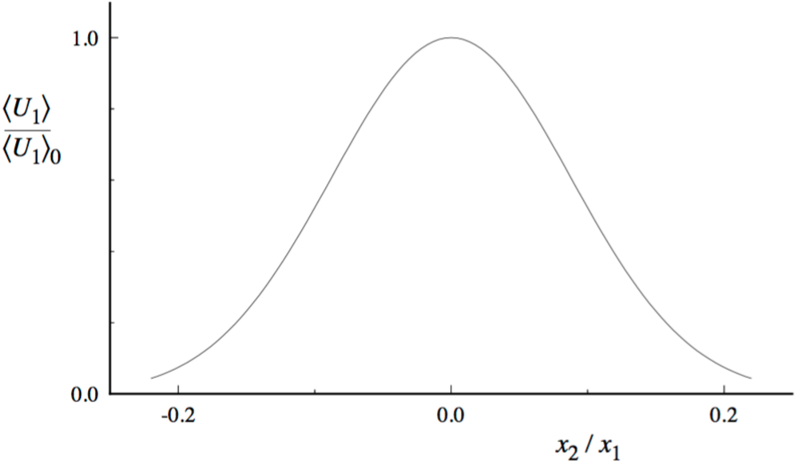
Random Nature of Turbulence





The left panel shows concentration in a turbulent jet, while the right shows the time history along the centerline (see Pope).

Random Nature of Turbulence



Normalized mean axial velocity in a turbulent jet (see Pope).

Random Nature of Turbulence

- The random behavior observed in the time series can appear to contradict what we know about fluids from classical mechanics.
- The Navier-Stokes equations are deterministic (*i.e.*, they give us an exact mathematical description of the evolution of a Newtonian fluid).
- Yet, as we have seen, turbulent flows are random.
- How do we resolve this inconsistency?

Random Nature of Turbulence

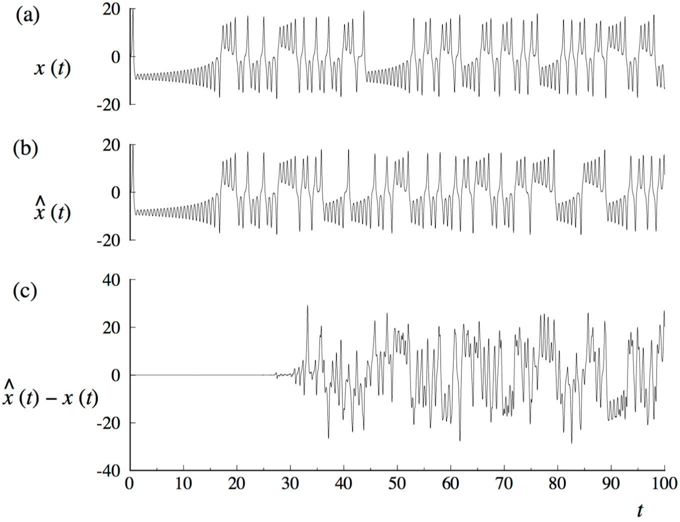
Question: Why the randomness?

- There are unavoidable perturbations (*e.g.*, initial conditions, boundary conditions, material properties, forcing, etc.) in turbulent flows.
- Turbulent flows and the Navier-Stokes equations are acutely sensitive to these perturbations.
- These perturbations do not fully explain the random nature of turbulence, since such small changes are present in laminar flows.
- However, the sensitivity of the flow field to these perturbations at large Re is much higher.

Random Nature of Turbulence

- This sensitivity to initial conditions has been explored extensively from the viewpoint of dynamical system. This is often referred to as chaos theory.
- The first work in this area was carried out by Lorenz (1963) in the areas of atmospheric turbulence and predictability. Perhaps you have heard the colloquial phrase, *the butterfly effect*.
- Lorenz studied a system with three state variables x, y, and z (see his paper or Pope for details). He ran one experiment with $x(0) = 1$ and another with $x = 1.000001$, while y and z were held constant.

Random Nature of Turbulence



Time history of the Lorenz equations.

Random Nature of Turbulence

- The work by Lorenz demonstrates the extreme sensitivity to initial conditions.
- The result of this sensitivity is that beyond some point, the state of the system cannot be predicted (*i.e.*, the limits of predictability).
- In the Lorenz example, even when the initial state is known to within 10^{-6}, predictability is limited to $t = 35$.

Random Nature of Turbulence

- In the Lorenz example, this behavior depends on the coefficients of the system. If a particular coefficient is less than some critical value, the solutions are stable. If, on the other hand, it exceeds that value, then the system becomes chaotic.
- This is similar to the Navier-Stokes equations, where solutions are steady for a sufficiently small Re, but turbulent if Re becomes large enough.

Random Nature of Turbulence

- We have seen that turbulent flows are random, but their histograms are apparently reproducible.
- As a consequence, turbulence is usually studied from a statistical viewpoint.