Environmental Fluid Dynamics: Lecture 7

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Overview

1. Atmospheric Thermodynamics: Evapotranspiration
   - Definition
   - Physics
   - Methods of Determination

2. Static Stability
   - Overview
   - Local Static Stability
   - Mixed Layer
   - Inversion
   - Non-Local Static Stability

Atmospheric Thermodynamics:

Evapotranspiration

Atmospheric Thermodynamics: Evapotranspiration

Evaporation

Conversion from liquid to vapor state

- free water surface
- moist soil surface
- leaves of living plants $\Rightarrow$ transpiration

Atmospheric Thermodynamics: Evapotranspiration

- Similar to molecular transfer of heat and momentum at a surface
- *Mass transfer* - occurring within first few molecular path lengths of the surface
- **Fick's Law**

  The net flux of mass in a given direction is proportional to the concentration gradient in that direction

Fick's Law

- The evaporation rate over a horizontal surface for a laminar flow is given by

$$E_0 = -\rho\alpha_w\frac{\partial q}{\partial z}$$

where

  - E_0 $[\mathrm{kg\ m^{-2}\ s^{-1}}]$ is the evaporation rate
  - ρ $[\mathrm{kg\ m^{-3}}]$ is total density
  - α_w $[\mathrm{m^2\ s^{-1}}]$ is molecular diffusivity of water.
  - q is specific humidity

  Typical values of α are

$$\alpha_w(0) = 21.2\times 10^{-6}\ \mathrm{m^2\ s^{-1}}$$
$$\frac{\alpha(T)}{\alpha_w(0)} = (1+0.007T)$$

Atmospheric Thermodynamics: Evapotranspiration

Turbulent flow

- mixing is dominated by "eddy motion" that is much more efficient
- phenomenological model

$$E = -\rho K_w \frac{\partial \overline{q}}{\partial z}$$

  where

  - K_w is the turbulent transfer coefficient ("eddy diffusivity"), which is much greater in magnitude than its molecular counterpart ($\sim 0.1 - 1 \ \mathrm{m}^2 \ \mathrm{s}^{-1}$)
  - K_w has limited applicability, is not constant (depends on T)

Consider a fully saturated soil surface, *i.e.*, moisture content is no longer a limiting factor:

Potential evaporation rate E_p

> *the maximum rate of evaporation for a given surface under a set of meteorological conditions*

For a unsaturated soil surface:

Actual evaporation rate depends on

- T_S
- turbulence: near-surface wind speed
- specific humidity
- stability
- plant physiology
- moisture content of soil

Methods for Determining: Direct Measurement

Lysimeter

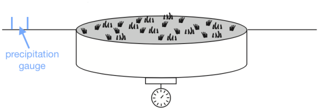


Figure 3.4 A weighing lysimeter sitting flush with the surface. The cylinder is filled with soil and vegetation similar to the surroundings.

From Davie (2008)

- Water mass balance on the soil

$$P = E + \Delta r + \Delta s$$

where

  - P is precipitation
  - E is evapotranspiration
  - Δr is runoff
  - Δs is storage

- Most have $\sim$ 1-6 m diameter and 1-2 m depth
- Basically, the change in storage of water is modeled by the change in weight of the soil

Evaporation Pan

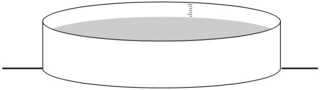


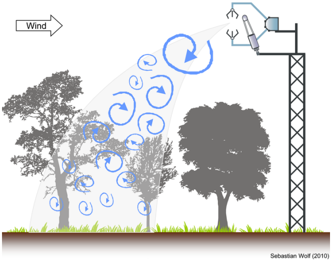
Figure 3.3 An evaporation pan. This sits above the surface (to lessen rain splash) and has either an instrument to record water depth or a continuous weighing device, to measure changes in volume.

From Davie (2008)

- Cylindrical pans (1-5 m diameter, 0.25-1 m depth)
- Measures free-water evaporation E_0 by monitoring water-level change in the pan
- E_0 is generally larger than E_t because evaporation in a catchment occurs over land where available water is in soil and potentially limited
- More appropriate for estimating water loss from lakes, ponds, and reservoirs

Methods for Determining: Direct Measurement

Eddy Covariance



From Wolfe (2010)

- Measures vertical transport of water vapor driven by convective motion with a sonic anemometer + IRGA or open path hygrometer
- Flux is instantaneously determined by sensing the properties of eddies as they pass through the sensor

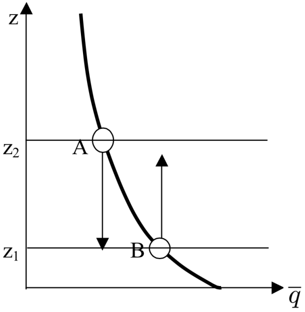
$$E = \rho \overline{w'q'}$$

where the bar is the average over some specified temporal window

- Let's relate these fluxes to gradients

Methods for Determining: Direct Measurement

Eddy Covariance



From Wolfe (2010)

- Specific humidity usually decreases with height
- Imagine parcel A is moved down, and parcel B is moved up, by some eddy
- For parcel A

$$w' < 0,\ q' < 0,\ w'q' > 0$$

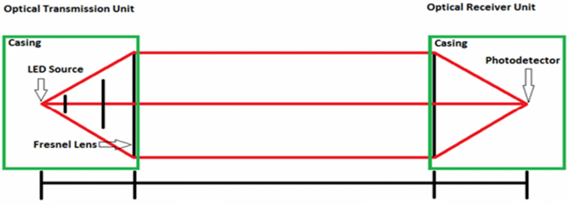
- For parcel B

$$w' > 0,\ q' > 0,\ w'q' > 0$$

Methods for Determining: Direct Measurement

Scintillometry

- Uses the concept that there are large variations of the index of refraction in the atmosphere that result from turbulent eddy motion
- Measures radiation intensity fluctuations from the laser source that result from humidity and temperature variations



Methods for Determining: Direct Measurement

Scintillometry

- Structure function

$$D_n(\vec{r}) = \overline{[n(\vec{x}) - n(\vec{x} + \vec{r}]^2}$$

  where $\vec{r}$ is a separation vector

- We can relate D_n to a constant called the structure parameter C_n^2:

$$C_n^2 = D_n / r^{(2/3)}$$

  where $r = |\vec{r}|$.

- This can further be related to C_T^2 and C_q^2 (more later in the course)

Methods for Determining: Direct Measurement

Scintillometry

- The intensity $I \propto C_n^2$
- Optical wavelengths (visible-near IR): $C_n^2 \rightarrow C_T^2$
- Radio wavelength (using RWS, expensive): $C_n^2 \rightarrow C_q^2$
- These can be used to determine H_S and H_L using Monin-Obukhov Similarity Theory (MOST), which implies that MOST is satisfied in the surface layer
- In this sense, the scintillometer approach is less direct since we obtain fluxes through empirical expressions

Methods for Determining: Direct Measurement

Scintillometry

- The Fresnel Length ($F = \sqrt{\lambda L}$) is a measure of the size of the most active eddy in a signal
- Small Aperture Scintillometers (SAS) are usually operated from $50\ \mathrm{m}$ to $500\ \mathrm{m}$
- Large Aperture Scintillometers (LAS) are usually operated from $500\ \mathrm{m}$ to $5\ \mathrm{km}$
- We get a spatial average, whereas other methods give point measurements

Energy Balance/Bowen Ratio

$$E_0 = \frac{H_L}{L_v}$$

where we showed already that

$$H_S = \frac{R_N - H_G}{1 + B^{-1}}$$
$$H_L = \frac{R_N - H_G}{1 + B}$$

and

$$B = \frac{H_S}{H_L} \approx \underbrace{\frac{\rho c_p K_H \dfrac{\partial \overline{\theta}}{\partial z}}{\rho L_v K_q \dfrac{\partial \overline{q}}{\partial z}}}_{K_H \sim K_q} \approx \frac{c_p}{L_v} \frac{\Delta \theta}{\Delta q}$$

Methods for Determining: Bulk Transfer Approach

Bulk Transfer Approach

- The bulk transfer formula for water vapor flux is

$$E_0 = \rho C_w U_r (q_0 - q_r)$$
$$H_0 = \rho c_p C_H (\theta_0 - \theta_r)$$

  where
  - C_w, C_H are the bulk transfer coefficients for water vapor and heat
  - U_r is velocity at some reference level
  - q_0, θ_0 are specific humidity and temperature at roughness height z_0
  - q_r, θ_r are specific humidity and temperature at some reference level
- Requires measurement of wind speed, temperature, and specific humidity
  - q_0 and θ_0 are very tough to measure at z_0

Bulk Transfer Approach

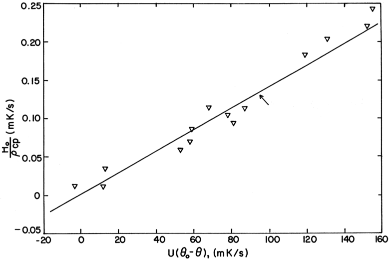
- Alternative form (usually good for water surfaces)

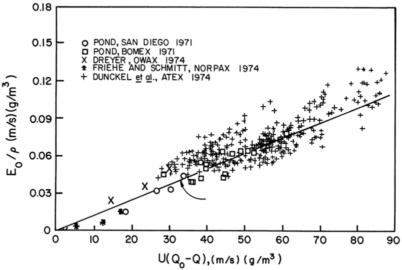
$$E_0 = C_w U_r (q_s - q_r)$$
$$H_0 = \rho c_p C_H U_r (\theta_s - \theta_r)$$

- Since z_o is so small for water surfaces, we can use surface values
- θ_s is available from satellite measurements
- q_s is determined from the saturation value at T_s
- Note: for neutral conditions, $C_w, C_H \sim 1.2 \times 10^{-3}$ (dependent on surface roughness, measurement height, and stability)

Methods for Determining: Bulk Transfer Approach

Bulk Transfer Approach





From Arya (2001)

Penman Approach

- A combination of energy balance and bulk transfer
- Penman (1948) derived a formula for evaporation over open water and saturated land

$$E_0 = \rho C_w U_r (q_0 - q_r^*) + E_a$$
$$E_a = \rho C_w U_r (q_r * - q_r)$$
$$H_0 = \rho c_p C_H U_r (T_0 - T_r)$$

where q_r^* is the saturation specific humidity and E_a is the "drying power" of air (related to advection)

Penman Approach

- If $C_w = C_H$

$$\frac{L_v E_0}{H_0} = \frac{\cancel{\rho} \cancel{C_w} L_v \cancel{U_r} (q_0 - q_r^*)}{\cancel{\rho} c_p \cancel{C_H} \cancel{U_r} (T_0 - T_r)} + \frac{L_v E_0}{H_0}$$

$$\frac{L_v E_0}{H_0} = \frac{L_v}{c_p} \frac{(q_0 - q_r^*)}{(T_0 - T_r)} + \frac{L_v E_0}{H_0} = \mathrm{BR}^{-1}$$

Recall, $q = 0.622 e/p$ - so

$$\mathrm{BR}^{-1} = \underbrace{\frac{0.622}{c_p} \frac{L_v}{p}}_{I} \underbrace{\frac{(e_0 - e_r^*)}{(T_0 - T_r)}}_{II} + \mathrm{BR}^{-1} \frac{E_a}{E_0}$$

Penman Approach

$$\mathrm{BR}^{-1} = \underbrace{\frac{0.622}{c_p}\frac{L_v}{p}}_{I}\underbrace{\frac{(e_0 - e_r^*)}{(T_0 - T_r)}}_{II} + \mathrm{BR}^{-1}\frac{E_a}{E_0}$$

- (I) $\gamma = \frac{c_p p}{0.622 L_v}$ = psychrometric constant
- (II) $\Delta = \frac{(e_0 - e_r^*)}{(T_0 - T_r)} \approx \frac{de_s}{d_T}$, valid when e_r^* is close to e_0 (i.e., a nearly saturated surface)
- Note: Δ is the slope of the saturation vapor pressure vs. temperature curve evaluated at $(T_0 + T_r)/2$

Penman Approach

- Thus,

$$\mathsf{BR}^{-1} = \frac{\Delta}{\gamma} + \mathsf{BR}^{-1}\frac{E_a}{e_0}$$
$$\mathsf{BR}(\frac{\Delta}{\gamma}) = 1 - \frac{E_a}{E_0} = \frac{E_0 - E_a}{E_0}$$
$$\boxed{\mathsf{BR} = \frac{\gamma}{\Delta}\left(\frac{E_0 - E_a}{E_0}\right)}$$

- Recall from the energy equation that

$$H_L = \frac{R_N - H_G}{1 + \mathsf{BR}} = E_0 L_v$$

such that

$$\mathsf{BR} = \frac{R_N - H_G}{E_o L_v} - 1$$

Penman Approach

- Equating yields

$$\left[\frac{\gamma}{\Delta}\left(\frac{E_0 - E_a}{E_0}\right) = \frac{R_N - H_G}{E_o L_v} - 1\right] E_0$$

$$\frac{\gamma}{\Delta}(E_0 - E_a) = \frac{R_N - H_G}{L_v} - E_0$$

$$E_0\left(1 + \frac{\gamma}{\Delta}\right) = \frac{R_N - H_G}{L_v} + E_a \frac{\gamma}{\Delta}$$

$$\boxed{E_0 = \frac{\Delta}{\gamma + \Delta}\frac{R_N - H_G}{L_v} + \underbrace{\frac{\gamma}{\gamma + \Delta}E_a}_{\text{III}}}$$

where (III) is departure from equilibrium in the atmosphere

Penman Approach

$$E_0 = \frac{\Delta}{\gamma + \Delta}\frac{R_N - H_G}{L_v} + \frac{\gamma}{\gamma + \Delta}E_a$$

- Need to measure:
    - soil heat flux
    - net radiation
    - temperature
    - humidity
    - wind speed
- Do not need surface temperature!

Penman Approach

$$E_0 = \frac{\Delta}{\gamma + \Delta}\frac{R_N - H_G}{L_v} + \frac{\gamma}{\gamma + \Delta}E_a$$

- To simplify, note that for a wide, very wet surface $e \to e_s$

$$E_0 = \frac{\Delta}{\gamma + \Delta}\frac{R_N - H_G}{L_v}$$

  this is called the equilibrium evapotranspiration
- Here, we only need to measure T and estimate $R_N - H_G$
- If valid, it is implied that

$$\mathsf{BR} = \frac{\gamma}{\Delta}$$

- Sensible heat flux (E_a negligible)

$$H_S \approx \frac{\gamma}{\gamma + \Delta}(R_N - H_G)$$

Priestly-Taylor Modification

$$H_L \approx \frac{\Delta}{\Delta + \gamma}(R_N - H_G)\alpha_{pt}$$
$$H_S \approx \frac{\gamma}{\gamma + \Delta}(R_N - H_G)(1 - \alpha_{pt})$$

where $\alpha_{pt} \sim 1.25$ for advection free conditions on a well waters surface (JAM,, 1982)

de Bruin and Holtslag Modification

Canopy with storage for non-saturated surface

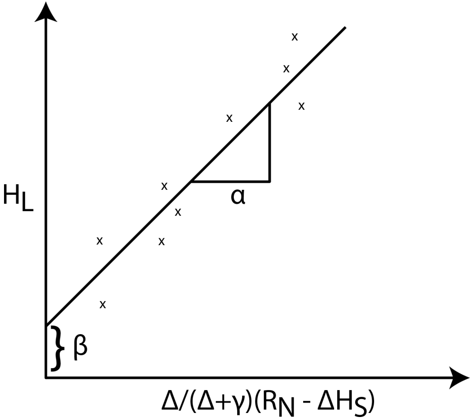
$$H_L \approx \alpha \frac{\Delta}{\Delta + \gamma}(R_N - \Delta H_s) + \beta$$

$$H_S \approx (1 - \alpha) \frac{\Delta}{\Delta + \gamma}(R_N - \Delta H_s) - \beta$$

where α depends on moisture status

This is almost identical to the LUMPS model (see paper)

LUMPS Model



- α and β are really just regression coefficients
- Recommended $\beta = 20\ \mathrm{W\ m^{-2}}$ (Holtslag and van Ulden 1983)
- For urban surface, $\beta = 3\ \mathrm{W\ m^{-2}}$ (Grimmond)

Static Stability

Static Stability

Arya Chapter 5.3

- Variations of temperature and humidity with height in PBL lead to density stratification
- As a consequence, an upward- or downward-moving air parcel will have a different density than its environment
- This leads to a buoyancy force that acts to accelerate/decelerate the vertical movement
- If the vertical movement is enhanced, the environment is **statically unstable**
- If the vertical movement is stopped, the environment is **statically stable**
- When the atmosphere exerts no buoyancy force, the environment is **neutral**

- Using Archimedes principle (idea that there is a balance between pressure and the weight of an body at equilibrium), the buoyancy force is

$$a_b = g\left(\frac{\rho - \rho_P}{\rho_P}\right)$$

and using the equation of state for moist air

$$a_b = g\left(\frac{T_v - T_{vp}}{T_v}\right)$$

where the subscript p refers to the parcel

Local Static Stability

- We can approximate a_b using the local gradient of virtual temperature

$$a_b \approxeq -\frac{g}{T_v}\left(\frac{\partial T_v}{\partial z} + \Gamma\right)\Delta z = -\frac{g}{T_v}\frac{\partial \theta_v}{\partial z}\Delta z$$

where

$$\theta_v = T_v \left(\frac{1000}{p}\right)^{\kappa}$$

is the virtual potential temperature, which allows for comparison of parcels with different pressures and moisture content - a very important parameter for stability determination

Local Static Stability

We define the static stability parameter s as

$$s = \frac{g}{T_v}\left(\frac{\partial \theta_v}{\partial z}\right)$$

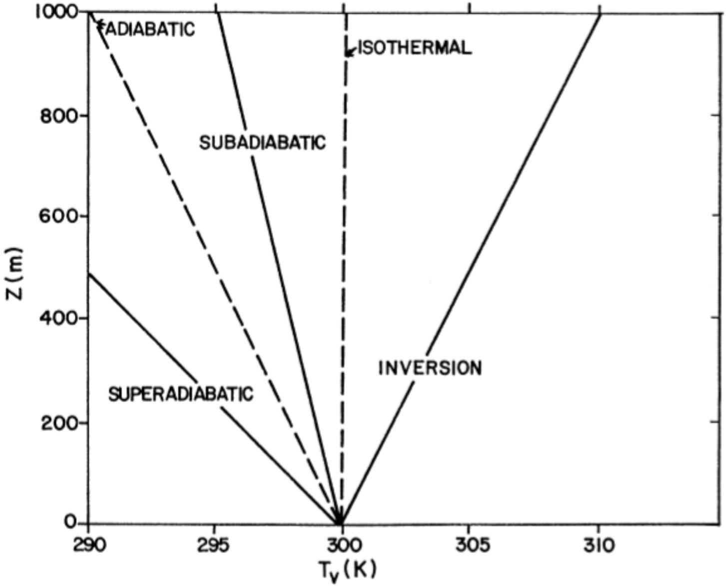
- Unstable: $s < 0$, $\partial\theta_v/\partial z < 0$, or $\partial T_v/\partial z < -\Gamma$
- Stable: $s > 0$, $\partial\theta_v/\partial z > 0$, or $\partial T_v/\partial z > -\Gamma$
- Neutral: $s = 0$, $\partial\theta_v/\partial z = 0$, or $\partial T_v/\partial z = -\Gamma$

Local Static Stability

Based on the virtual temperature gradient or lapse rate(LR), relative to the adiabatic lapse rate Γ, layers are characterized as

- Superadiabatic: LR $> \Gamma$
- Adiabatic: LR $= \Gamma$
- Subadiabatic: LR $< \Gamma$
- Isothermal: LR $= 0$
- Inversion: LR > 0

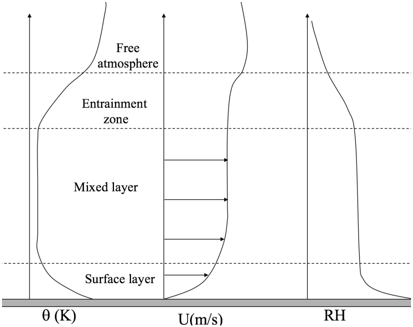
Local Static Stability



Local Static Stability

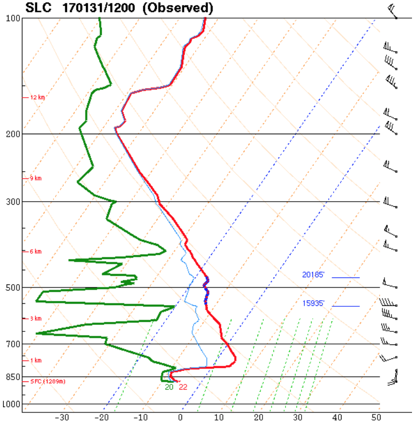
- The local view of static stability is limited and flawed
- This is especially true when used as a measure of turbulent mixing and diffusion
- The bulk of the CBL is a mixed layer ($\partial\theta_v/\partial z \approx 0$, or slightly positive)
- This would leave you to think this is a neutral or slightly stable layer according to local static stability theory
- However, the CBL has every attribute of an unstable layer: upward heat flux, strong mixing, large thickness
- Thus, s is a poor metric for parcels that undergo large displacements from equilibrium - need a new idea not based on local gradients

Mixed Layer



- layer where significant mixing occurs
- most properties are constant

Mixed Layer



- θ_v increases with height (locally stable)
- vertical mixing is inhibited, pollution can increase
- examples: surface cooling in SL valley, cold pool,, warm sea breeze blowing over cool land

Inversion

Yuck!

Non-Local Static Stability

- Enter non-local static stability (Stull 1991)
- Need soundings of environment over a deep layer up to place where vertical motions are irrelevant (strong inversion, tropopause)
- Stability is determined by displacing parcels from all possible locations in the domain
- Thus the stability is based on parcel buoyancy and not local lapse rates
- Parcel buoyancy at any level is determined by the difference in virtual temperatures of the parcel and environment

Non-Local Static Stability

- Unstable: regions where parcels can enter and transit under their own buoyancy - note: parcels need not traverse the entire region
- Stable: regions of superadiabatic LR that are not unstable
- Neutral: regions of adiabatic lapse rates that are not unstable
- Unknown: Top or bottom portions of the sounding that appear stable or neutral, but do not end at a material surface (ground, inversion, etc) - because area above or below is unknown and could provide positive buoyancy

Non-Local Static Stability