Environmental Fluid Dynamics: Lecture 6

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Overview

Atmospheric Thermodynamics:

Water Vapor

Atmospheric Thermodynamics: Water Vapor

- So far we have discussed water vapor in the air through its vapor pressure e
- We included its effects on density through the virtual temperature correction
- There are many ways in which to describe the amount of water vapor present in the atmosphere - so we will discuss various moisture paramters
- What happens when water vapor condenses? We will cover that, too.

Moisture Parameters: Latent Heat

Latent Heat of Vaporization/Evaporation

The heat required by a unit mass of material to convert it from the liquid to gas phase without a change in temperature

- At 1 atm and $100\ ^\circ\mathrm{C}$ (boiling point of water), $L_v = 2.25 \times 10^6\ \mathrm{J\ kg^{-1}}$
- The Latent Heat of Condensation has the same value, but heat is released when changing from vapor to liquid

Moisture Parameters: Mixing Ratio

Mixing Ratio

The amount of water vapor in a volume of air expressed as the ratio of the mass of water vapor m_v to the mass of dry air m_d

$$r \equiv \frac{m_v}{m_d}$$

- Usually expressed as $[\mathrm{g}_v/\mathrm{kg}_d]$
- Dimensionless for numerical computations $[\mathrm{kg}_v/\mathrm{kg}_d]$
- Ranges from a few $\mathrm{g}\ \mathrm{kg}^{-1}$ in midlatitudes to $20\ \mathrm{g}/\mathrm{kg}^{-1}$ in the tropics
- In the absence of condensation/evaporation, an air parcel's r is constant (conserved)

Moisture Parameters: Mixing Ratio

- We can relate mixing ratio r to T_v
- Let's recall the notion of partial pressures. The partial pressure of a gas is proportional to the number of moles of that gas present in the mixture

$$e = \frac{n_v}{n_d + n_v} p = \frac{\frac{m_v}{M_w}}{\frac{m_d}{M_d} + \frac{m_v}{M_w}} p = \frac{m_v}{\frac{m_d M_w}{M_d} + \frac{m_v M_w}{M_w}} p = \frac{\frac{m_v}{m_d}}{\epsilon \frac{m_d}{m_d} + \frac{m_v}{m_d}} p$$

$$\boxed{e = \frac{r}{r + \epsilon} p}$$

where, recall, $\epsilon = R_d / R_v = M_w / M_d = 0.622$

Moisture Parameters: Mixing Ratio

$$e = \frac{r}{r+\epsilon}p$$

- Recall from Lecture 4 that

$$T_v \equiv \frac{T}{1 - \frac{e}{p}(1-\epsilon)}$$

- Replace e/p with the expression derived on the previous slide

$$T_v == \frac{T}{1 - \frac{r}{r+\epsilon}(1-\epsilon)} = \frac{T}{\frac{r+\epsilon-r+r\epsilon}{r+\epsilon}} = T\frac{r+\epsilon}{\epsilon(1+r)}$$

Note that $r \ll 1$, so we can approximate $(1+r)^{-1} \sim (1-r)$

Moisture Parameters: Mixing Ratio

- Substitution yields

$$T_v = T\left[\left(\frac{r}{\epsilon}+1\right)(1-r)\right] = T\left[\frac{r}{\epsilon} - \cancel{\frac{r^2}{\epsilon}} + 1 - r\right]$$

$$\simeq T\left[1 + r\left(\frac{1}{\epsilon}-1\right)\right] = T\left[1 + r(1.61-1)\right]$$

$$\boxed{T_v \simeq T(1+0.61r)}$$

This is a useful expression to obtain T_v with just T and the mixing ratio

- Remember that virtual temperature is the temperature that dry air would need to attain in order to have the same density as moist air at the same pressure

Moisture Parameters: Specific Humidity

Specific Humidity

The amount of water vapor in a volume of air expressed as the ratio of the mass of water vapor m_v to the total mass of the air $(m_d + m_v)$

$$q \equiv \frac{m_v}{(m_d + m_v)} = \frac{m_v/m_d}{m_d/m_d + m_v/m_d} = \frac{r}{r+1}$$

- Since r is usually only a few $\%$, then r and q do not differ greatly

Moisture Parameters: Specific Humidity

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Moisture Parameters: Absolute Humidity

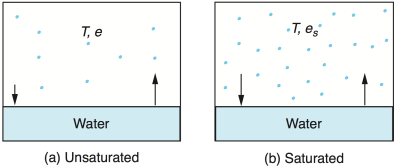
Absolute Humidity

The mass of water vapor m_v per unit volume of moist air

$$\rho_v = \frac{m_v}{\cancel{V}}$$

- Also referred to as vapor density
- Because ρ_v is not conservative w.r.t. adiabatic expansion or compression, it is not commonly used in atmospheric sciences

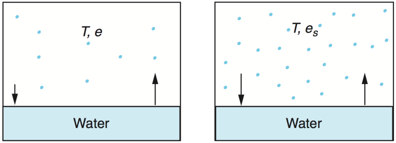
Moisture Parameters: Saturation Vapor Pressure



From Wallace and Hobbs (2006)

- Consider a small closed box whose floor is covered by pure water at temperature T
- Let's assume that the air is initially completely dry
- Evaporation begins and the number of water molecules in the box (thus e) increases
- As e gets larger, the fast water condense back into liquid form

Moisture Parameters: Saturation Vapor Pressure



(a) Unsaturated (b) Saturated

From Wallace and Hobbs (2006)

- If the rate of condensation $>$ rate of evaporation, then the box is **unsaturated** at T
- If the rate of condensation $=$ rate of evaporation, then the box is **saturated** at T
- The pressure exerted by the water vapor int he box is called the **saturation vapor pressure** e_s

Moisture Parameters: Saturation Vapor Pressure

Saturation Vapor Pressure Deficit

The difference between the saturated vapor pressure at a particular temperature and the water vapor pressure

$$VPD = e_s(T) - e$$

- VPD is sometimes referred to as the "drying power" of air in ecology problems

Moisture Parameters: Saturation Vapor Pressure

A quick aside

- You might hear phrases like

> *The air is saturated with water vapor*
> *Warm air holds more wv than cold air*
> *The air cannot hold more wv*

These suggest that air absorbs water vapor like a sponge.

Wrong! Stop That!

Moisture Parameters: Saturation Vapor Pressure

A quick aside

- Recall Dalton's Law of Partial Pressures - the total pressure is equal to the partial pressure of each constituent
- Thus, the phase change of water between liquid and vapor form is independent of air
- Water vapor that is in equilibrium with water at T should more appropriately called the **equilibrium vapor pressure**

Moisture Parameters: Saturation Vapor Pressure

- Recall Dalton's Law of Partial Pressures - the total pressure is equal to the partial pressure of each constituent
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Moisture Parameters: Saturation Vapor Pressure

- How do we find $e_s(T)$? By using the Clausius–Clapeyron equation
- It relates the saturation vapor pressure to temperature
- From Maxwell's Equations (2$^{\text{nd}}$ Law of Thermodynamics)

$$\frac{de_s}{e_s} = \frac{L_v}{R_v}\frac{dT}{T^2}$$

where

- e_s is saturation vapor pressure
- L_v is the latent heat of vaporization
- T is temperature
- R_v is the gas constant for water vapor

Moisture Parameters: Saturation Vapor Pressure

- We integrate from state 1 to state 2

$$\int_{S1}^{S2} \frac{de_s}{e_s} = \int_{S1}^{S2} \frac{L_v}{R_v} \frac{dT}{T^2}$$

$$\ln\left(\frac{e_s(S2)}{e_s(S1)}\right) = \frac{L_v}{R_v}\left(\frac{1}{T(S1)} - \frac{1}{T(S2)}\right)$$

$$ln\left(\frac{e_s}{e_{s0}}\right) = \frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)$$

  where we let $S1$ represent a reference state (denoted with subscript 0)

- $T(S1) = T_0 = 273.1\ \mathrm{K}$
- Experimental data has shown $e_s(S1) = e_{s0} = 6.11\ \mathrm{hPa}$

Moisture Parameters: Saturation Vapor Pressure

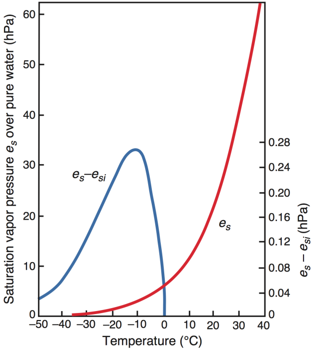
- Rearranging gives e_s for any T

$$e_s = 6.11 \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273.1} - \frac{1}{T}\right)\right]$$

- From this, we can solve for the vapor pressure e from measurements of RH and T by way of

$$RH = 100\frac{e}{e_s}$$

Moisture Parameters: Saturation Vapor Pressure



From Wallace and Hobbs (2006)

- The evaporation rate increases with increasing temperature
- Thus, e_s increase with increasing temperature
- Its magnitude only depends on temperature

Moisture Parameters: Saturation Mixing Ratio

Saturation Mixing Ratio

The amount of water vapor in a volume of air that is saturated, expressed as the ratio of the mass of water vapor m_{vs} *to the mass of dry air* m_d

$$r_s \equiv \frac{m_{vs}}{m_d}$$

- Both water vapor and dry air obey the ideal gas law

$$r_s = \frac{\rho'_{vs}}{\rho'_d} = \frac{\dfrac{e_s}{R_v T}}{\dfrac{(p - e_s)}{R_d T}} = \frac{R_d}{R_v}\frac{e_s}{p - e_s} = \epsilon \frac{e_s}{p - e_s}$$

Moisture Parameters: Saturation Mixing Ratio

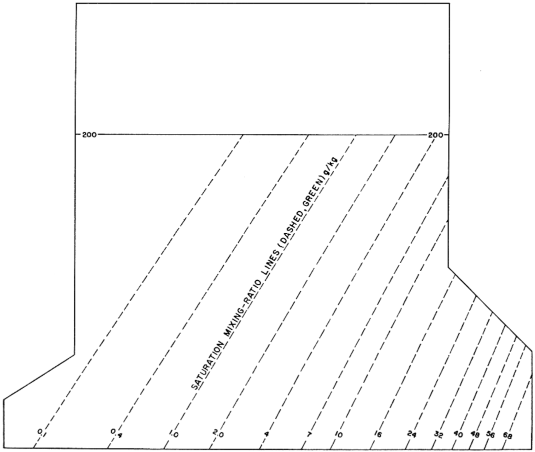
- Given typical values in the atmosphere, $p \gg e_s$, so

$$r_s = \epsilon \frac{e_s}{p - e_s} \simeq \epsilon \frac{e_s}{p}$$

$$\boxed{r_s \simeq 0.622 \frac{e_s}{p}}$$

- Thus, r_s is inversely proportional to total pressure at a given temperature
- Since $e_s = e_s(T)$, then $r_s = r_s(p, T)$
- This can be seen on a skew T-$\ln$ p chart

Moisture Parameters: Saturation Mixing Ratio



- For constant T, r_s increases with decreasing p
- For constant p, r_s increases with increasing T

Moisture Parameters: Relative Humidity and Dew Point

Relative Humidity

The ratio of the mixing ratio to the saturation mixing ratio at the same temperature and pressure

$$RH \equiv 100\frac{r}{r_s} \simeq 100\frac{q}{q_s} \simeq 100\frac{e}{e_s}$$

Dew Point Temperature

The temperature to which air must be cooled at constant pressure for it become saturated w.r.t water

$$T_d = T(r_s = r)$$

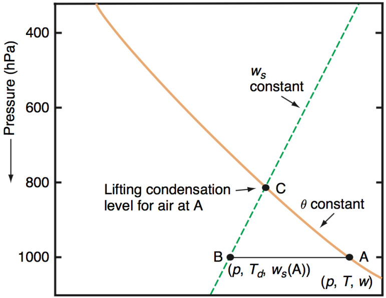
$$T_d \simeq T - \frac{100 - RH}{5}$$

Moisture Parameters: Lifting Condensation Level

Lifting Condensation Level (LCL)

The level to which a moist unsaturated air parcel can be lifted adiabatically before becoming saturated

- As the parcel rises, r and θ remain constant while r_s decreases until it equals r (at the LCL)
- The LCL is located where dry adiabat and r_s line intersect



Saturated Adiabatic Processes

- When an air parcel rises in the atmosphere, T decreases with increasing height until it becomes saturated
- Condensation of liquid water occurs as the parcel is lifted further, which releases latent heat
- Thus, the lapse rate of the rising parcel is reduced

Saturated Adiabatic Processes

- If all the condensation remains in the air parcel, the process is considered reversible and thus adiabatic
- Although latent heat is released, as long as it remains within the confines of the air parcel then the parcel underwent a **saturated adiabatic process**
- If the condensation falls out of the parcel then the process is irreversible because condensation carries heat - so not really adiabatic
- In this case, it is called a **pseudoadiabatic process** (in practice the saturated adiabatic and pseudoadiabtic lapse rates are approximately the same)

Saturated Adiabatic Lapse Rate

- We won't derive this in class, but for posterity here is the saturated adiabatic lapse rate

$$\Gamma_s \simeq \frac{\Gamma_d}{1 + \frac{L_v}{c_p}\left(\frac{\partial r_s}{dT}\right)_p}$$

- Note that the denominator is > 1, so $\Gamma_d > \Gamma_s$, which agrees with our expectation
- Values range from 4 K kg^{-1} near the ground in warm moist air to $6\text{-}7 \text{ K kg}^{-1}$ in the middle portion of the troposphere

Saturated Adiabatic Lapse Rate

- We won't derive this in class, but for posterity here is the equivalent potential temperature θ_e
- Just as dry adiabats are lines of constant θ, moist adiabats are lines of constant θ_e

$$\theta_e \simeq \theta \exp\left(\frac{L_v r_s}{c_p T}\right)$$

Saturated Adiabatic Processes