Environmental Fluid Dynamics: Lecture 6

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Atmospheric Thermodynamics: Water Vapor
Moisture Parameters
Saturated Adiabatic Processes
Atmospheric Thermodynamics: Water Vapor
So far we have discussed water vapor in the air through its vapor pressure $e$

We included its effects on density through the virtual temperature correction

There are many ways in which to describe the amount of water vapor present in the atmosphere - so we will discuss various moisture parameters

What happens when water vapor condenses? We will cover that, too.
Latent Heat of Vaporization/Evaporation

The heat required by a unit mass of material to convert it from the liquid to gas phase without a change in temperature

- At 1 atm and 100 °C (boiling point of water),
  \[ L_v = 2.25 \times 10^6 \text{ J kg}^{-1} \]
- The Latent Heat of Condensation has the same value, but heat is released when changing from vapor to liquid
Moisture Parameters: Mixing Ratio

Mixing Ratio

The amount of water vapor in a volume of air expressed as the ratio of the mass of water vapor $m_v$ to the mass of dry air $m_d$

$$r \equiv \frac{m_v}{m_d}$$

- Usually expressed as $[g_v/\text{kg}_d]$
- Dimensionless for numerical computations $[\text{kg}_v/\text{kg}_d]$
- Ranges from a few $g \text{ kg}^{-1}$ in midlatitudes to 20 $g/\text{kg}^{-1}$ in the tropics
- In the absence of condensation/evaporation, an air parcel’s $r$ is constant (conserved)
Moisture Parameters: Mixing Ratio

- We can relate mixing ratio $r$ to $T_v$
- Let’s recall the notion of partial pressures. The partial pressure of a gas is proportional to the number of moles of that gas present in the mixture

$$ e = \frac{n_v}{n_d + n_v} p = \frac{m_v}{M_w} p = \frac{m_v}{m_d + m_v} p = \frac{1}{\epsilon} \frac{m_v}{m_d} p $$

$$ e = \frac{r}{r + \epsilon} p $$

where, recall, $\epsilon = R_d/R_v = M_w/M_d = 0.622$
Moisture Parameters: Mixing Ratio

\[ e = \frac{r}{r + \epsilon p} \]

- Recall from Lecture 4 that
  \[ T_v \equiv \frac{T}{1 - \frac{e}{p}(1 - \epsilon)} \]

- Replace \( e/p \) with the expression derived on the previous slide
  \[ T_v = \frac{T}{1 - \frac{r}{r + \epsilon(1 - \epsilon)}} = \frac{T}{\frac{r + \epsilon - r + r\epsilon}{r + \epsilon}} = T \frac{r + \epsilon}{\epsilon(1 + r)} \]

Note that \( r \ll 1 \), so we can approximate \( (1 + r)^{-1} \sim (1 - r) \)
Moisture Parameters: Mixing Ratio

- Substitution yields

\[ T_v = T \left[ \left( \frac{r}{\epsilon} + 1 \right) (1 - r) \right] = T \left[ \frac{r}{\epsilon} - \frac{r^2}{\epsilon} + 1 - r \right] \]

\[ \simeq T \left[ 1 + r \left( \frac{1}{\epsilon} - 1 \right) \right] = T \left[ 1 + r(1.61 - 1) \right] \]

\[ T_v \simeq T(1 + 0.61r) \]

This is a useful expression to obtain \( T_v \) with just \( T \) and the mixing ratio.

- Remember that virtual temperature is the temperature that dry air would need to attain in order to have the same density as moist air at the same pressure.
Moisture Parameters: Specific Humidity

**Specific Humidity**

*The amount of water vapor in a volume of air expressed as the ratio of the mass of water vapor* $m_v$ *to the total mass of the air* $(m_d + m_v)$

$$q \equiv \frac{m_v}{(m_d + m_v)} = \frac{m_v/m_d}{m_d/m_d + m_v/m_d} = \frac{r}{r + 1}$$

- Since $r$ is usually only a few %, then $r$ and $q$ do not differ greatly
Moisture Parameters: Specific Humidity

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Moisture Parameters: Absolute Humidity

**Absolute Humidity**

The mass of water vapor \( m_v \) per unit volume of moist air

\[
\rho_v = \frac{m_v}{V}
\]

- Also referred to as vapor density
- Because \( \rho_v \) is not conservative w.r.t. adiabatic expansion or compression, it is not commonly used in atmospheric sciences
Moisture Parameters: Saturation Vapor Pressure

Consider a small closed box whose floor is covered by pure water at temperature $T$

Let’s assume that the air is initially completely dry

Evaporation begins and the number of water molecules in the box (thus $e$) increases

As $e$ gets larger, the fast water condense back into liquid form

From Wallace and Hobbs (2006)
Moisture Parameters: Saturation Vapor Pressure

- If the rate of condensation > rate of evaporation, then the box is **unsaturated** at $T$
- If the rate of condensation = rate of evaporation, then the box is **saturated** at $T$
- The pressure exerted by the water vapor in the box is called the **saturation vapor pressure** $e_s$

From Wallace and Hobbs (2006)
Saturation Vapor Pressure Deficit

The difference between the saturated vapor pressure at a particular temperature and the water vapor pressure

\[ VPD = e_s(T) - e \]

- \( VPD \) is sometimes referred to as the “drying power” of air in ecology problems.
Moisture Parameters: Saturation Vapor Pressure

A quick aside

- You might hear phrases like

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"The air is saturated with water vapor
Warm air holds more wv than cold air
The air cannot hold more wv"
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These suggest that air absorbs water vapor like a sponge.

Wrong! Stop That!
A quick aside

- Recall Dalton’s Law of Partial Pressures - the total pressure is equal to the partial pressure of each constituent.
- Thus, the phase change of water between liquid and vapor form is independent of air.
- Water vapor that is in equilibrium with water at $T$ should more appropriately called the *equilibrium vapor pressure*. 
Moisture Parameters: Saturation Vapor Pressure

- Recall Dalton’s Law of Partial Pressures - the total pressure is equal to the partial pressure of each constituent
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Moisture Parameters: Saturation Vapor Pressure

- How do we find $e_s(T)$? By using the Clausius–Clapeyron equation
- It relates the saturation vapor pressure to temperature
- From Maxwell’s Equations (2\textsuperscript{nd} Law of Thermodynamics)

$$\frac{de_s}{e_s} = \frac{L_v}{R_v} \frac{dT}{T^2}$$

where
- $e_s$ is saturation vapor pressure
- $L_v$ is the latent heat of vaporization
- $T$ is temperature
- $R_v$ is the gas constant for water vapor
Moisture Parameters: Saturation Vapor Pressure

- We integrate from state 1 to state 2

\[
\int_{S_1}^{S_2} \frac{d e_s}{e_s} = \int_{S_1}^{S_2} \frac{L_v}{R_v} \frac{dT}{T^2}
\]

\[
\ln \left( \frac{e_s(S_2)}{e_s(S_1)} \right) = \frac{L_v}{R_v} \left( \frac{1}{T(S_1)} - \frac{1}{T(S_2)} \right)
\]

\[
\ln \left( \frac{e_s}{e_{s0}} \right) = \frac{L_v}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right)
\]

where we let \( S_1 \) represent a reference state (denoted with subscript 0)

- \( T(S_1) = T_0 = 273.1 \) K

- Experimental data has shown \( e_s(S_1) = e_{s0} = 6.11 \) hPa
Moisture Parameters: Saturation Vapor Pressure

• Rearranging gives $e_s$ for any $T$

$$e_s = 6.11 \exp \left[ \frac{L_v}{R_v} \left( \frac{1}{273.1} - \frac{1}{T} \right) \right]$$

• From this, we can solve for the vapor pressure $e$ from measurements of $RH$ and $T$ by way of

$$RH = 100 \frac{e}{e_s}$$
Moisture Parameters: Saturation Vapor Pressure

- The evaporation rate increases with increasing temperature
- Thus, $e_s$ increase with increasing temperature
- Its magnitude only depends on temperature

From Wallace and Hobbs (2006)
Saturation Mixing Ratio

The amount of water vapor in a volume of air that is saturated, expressed as the ratio of the mass of water vapor $m_{vs}$ to the mass of dry air $m_d$

$$r_s \equiv \frac{m_{vs}}{m_d}$$

- Both water vapor and dry air obey the ideal gas law

$$r_s = \frac{\rho'_{vs}}{\rho'_d} = \frac{e_s}{R_v T} = \frac{R_d}{R_v} \frac{e_s}{p - e_s} = \epsilon \frac{e_s}{p - e_s}$$
• Given typical values in the atmosphere, $p \gg e_s$, so

$$r_s = \epsilon \frac{e_s}{p - e_s} \simeq \frac{\epsilon e_s}{p}$$

$$r_s \simeq 0.622 \frac{e_s}{p}$$

• Thus, $r_s$ is inversely proportional to total pressure at a given temperature

• Since $e_s = e_s(T)$, then $r_s = r_s(p, T)$

• This can be seen on a skew T-ln p chart
Moisture Parameters: Saturation Mixing Ratio

- For constant $T$, $r_s$ increases with decreasing $p$
- For constant $p$, $r_s$ increases with increasing $T$
Relative Humidity

The ratio of the mixing ratio to the saturation mixing ratio at the same temperature and pressure

\[ RH \equiv 100 \frac{r}{r_s} \simeq 100 \frac{q}{q_s} \simeq 100 \frac{e}{e_s} \]

Dew Point Temperature

The temperature to which air must be cooled at constant pressure for it become saturated w.r.t water

\[ T_d = T(r_s = r) \]

\[ T_d \simeq T - \frac{100 - RH}{5} \]
Moisture Parameters: Lifting Condensation Level

**Lifting Condensation Level (LCL)**

*The level to which a moist unsaturated air parcel can be lifted adiabatically before becoming saturated*

- As the parcel rises, $r$ and $\theta$ remain constant while $r_s$ decreases until it equals $r$ (at the LCL)
- The LCL is located where dry adiabat and $r_s$ line intersect
Saturated Adiabatic Processes

- When an air parcel rises in the atmosphere, $T$ decreases with increasing height until it becomes saturated.
- Condensation of liquid water occurs as the parcel is lifted further, which releases latent heat.
- Thus, the lapse rate of the rising parcel is reduced.
Saturated Adiabatic Processes

- If all the condensation remains in the air parcel, the process is considered reversible and thus adiabatic.
- Although latent heat is released, as long as it remains within the confines of the air parcel then the parcel underwent a **saturated adiabatic process**.
- If the condensation falls out of the parcel then the process is irreversible because condensation carries heat - so not really adiabatic.
- In this case, it is called a **pseudoadiabatic process** (in practice the saturated adiabatic and pseudoadiabatic lapse rates are approximately the same).
• We won’t derive this in class, but for posterity here is the saturated adiabatic lapse rate

\[ \Gamma_s \simeq \frac{\Gamma_d}{1 + \frac{L_v}{c_p} \left( \frac{\partial r_s}{dT} \right)_p} \]

• Note that the denominator is > 1, so \( \Gamma_d > \Gamma_s \), which agrees with our expectation

• Values range from 4 K kg\(^{-1}\) near the ground in warm moist air to 6-7 K kg\(^{-1}\) in the middle portion of the troposphere
Saturated Adiabatic Lapse Rate

• We won’t derive this in class, but for posterity here is the equivalent potential temperature $\theta_e$

• Just as dry adiabats are lines of constant $\theta$, moist adiabats are lines of constant $\theta_e$

\[ \theta_e \simeq \theta \exp \left( \frac{L_v r_s}{c_p T} \right) \]
Saturated Adiabatic Processes