Environmental Fluid Dynamics: Lecture 4

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Overview

Atmospheric Thermodynamics

Numerous factors influence the vertical distribution of air temperature in the planetary boundary layer (PBL).

Examples:

- Type of air mass (and its temperature) overlying the PBL - depends on large-scale systems

- Thermal characteristics of surface/sub-surface - affects diurnal range

- Net radiation the surface and its vertical distribution - determines warming/cooling of surface and PBL

- Sensible heat flux at the surface and its vertical distribution - determines rate of warming/cooling of air due to flux convergence/divergence

Numerous factors influence the vertical distribution of air temperature in the PBL.

Examples:

- Latent heat fluxes during evaporation and condensation at the surface and in the air - directly influences surface and air temp

- Advection of warm of cold air as a function of height in the PBL

- Depth of PBL - confines turbulent exchanges over a particular distance

Likewise, numerous factors influence the vertical distribution of moisture in the PBL.

Examples:

- Specific humidity of air mass overlying the PBL
- Surface type, temperature, and availability of moisture for evaporation/transpiration
- The rate of evapotranspiration or condensation at the surface and vertical distribution of water vapor flux

Likewise, numerous factors influence the vertical distribution of moisture in the PBL.

Examples:

- Variation of horizontal water vapor advection with height
- Mean vertical motion in PBL - possibilities of cloud formation and precipitation processes
- The depth of the PBL

Goal: introduce fundamental ideas and relationships in thermodynamics and apply them to situations in the atmosphere

- gas laws
- hydrostatic equation
- first law of thermodynamics
- second law of thermodynamics
- entropy

Atmospheric Thermodynamics: Gas Laws

- Pressure, volume, and temperature of a material are described by an **equation of state**
- All gases generally follows the same equation of state - the **ideal gas law**
- We will assume that atmospheric gases obey this law

$$pV = mRT$$

where

- p = pressure (Pa)
- V = volume (m^3)
- m = mass (kg)
- R = gas constant (depends on gas)
- T = absolute temperature (K)

- The ideal gas law

$$pV = mRT$$

- Density $\rho = m/V$, so we can rewrite the ideal gas law as

$$p = \rho RT$$

- We can also introduce specific volume $\alpha = 1/\rho$, which is the volume occupied by unit mass of the gas

$$p\alpha = RT$$

$$pV = mRT$$

- If T is constant, we get Boyle's Law

  > *if the temperature of a fixed mass of gas is held constant, the volume of the gas is inversely proportional to its pressure*

- Changes that occur to a body's physical state under constant temperature are called *isothermal*

$$pV = mRT$$

- If m is fixed and p is constant, we get Charles' First Law

  for a fixed mass of gas at constant pressure, the volume of the gas is directly proportional to its absolute temperature

- If m is fixed and V is constant, we get Charles' Second Law

  for a fixed mass of gas held within a fixed volume, the pressure of the gas is proportional to its absolute temperature

- Let's define a kilogram-molecular weight (kilomole; kmol) as the molecular mass M expressed in kilograms
- Example: the molecular weight of water is 18.015, so $1 \mathrm{\ kmol} = 18.015 \mathrm{\ kg}$
- The number of moles n in mass m [kg] is given by

$$n = \frac{m}{M}$$

Atmospheric Thermodynamics: Gas Laws

- Masses contained in 1 mol of different substances have same ratios to each other as the molecular weights of the substances
- Thus, 1 mol of a subtance contains same numbers of molecules as 1 mol of another substance
- *i.e.*, the number of molecules in 1 mol of a substance is a universal constant
- *Avagadro's number* $= 6.022 \times 10^{23}$ per mole
- *Avagadro's hypothesis*

  Gases containing the same number of molecules occupy the same volumes at the same temperature and pressure

Dry Air

- We can use the apparent molecular weight of dry air (total mass of the constituent gases in dry air divided by the total number of moles of the constituent gases)

$$M_d = \frac{\sum_i m_i}{\sum_i \frac{m_i}{M_i}} = 29 \text{ kg kmol}^{-1}$$

to obtain the gas constant for dry air

$$R_d = \frac{R_u}{M_d} = \frac{8314}{29} = 287 \text{ J K}^{-1} \text{ kg}^{-1}$$

Atmospheric Thermodynamics: Gas Laws

Dry Air

- Why $29 \ \text{kg} \ \text{kmol}^{-1}$ for dry air?

Component	Volume Ratio	M_i	Molecular Mass
Oxygen	0.2095	32.00	6.704
Nitrogen	0.7809	28.02	21.88
Carbon Dioxide	3×10^{-4}	44.01	0.013
Hydrogen	5×10^{-7}	2.02	0
Argon	9.3×10^{-3}	39.94	0.373
Neon	1.8×10^{-5}	20.18	0
Helium	5×10^{-6}	4.00	0
Krypton	1×10^{-4}	83.8	0
Xenon	9×10^{-8}	131.29	0
Apparent Molecular Weight M_d			**28.97**

Water Vapor

- We can apply to individual components of air, for instance water vapor

$$e\alpha_v = R_v T$$

where

  - e = water vapor pressure
  - α_v = specific volume of water vapor
  - R_v is the gas constant for water vapor

- Since molecular weight of water is $M_w = 18.016$, then

$$R_v = \frac{R_u}{M_w} = \frac{8314}{18.016} = 461.5 \text{ J K}^{-1} \text{ kg}^{-1}$$

Atmospheric Thermodynamics: Gas Laws

- We can combine the two expressions to show that

$$\frac{R_d}{R_v} = \frac{M_w}{M_d} \equiv \epsilon = 0.622$$

- Since air is a mixture of gases, it follows Dalton's Law of Partial Pressures

  the total pressure exerted by a mixture of gases that do not interact chemically is equal to the sum of the partial pressures of the gases

  $$p = \sum_i p_i$$

Moist Air

- Moist air has a smaller molecular weight than dry air

- Thus, moist air will have a larger gas constant than dry air

- it is inconvenient, however, to use the gas constant for moist air because it depends on amount of water vapor in air (variable, hard to measure)

- Optional idea is to use R_d with an ad-hoc temperature variable that accounts for moisture effects

- We will call this **virtual temperature**

Virtual Temperature

- The density of moist air is given by

$$\rho = \frac{m_d + m_v}{V} = \rho'_d + \rho'_v$$

  where

  - ρ'_d = density that the mass of dry air would have if occupied all of the volume
  - ρ'_v = density that the mass of moist air would have if occupied all of the volume
  - We call these partial densities

- Note: moist air is less dense than dry air!

Virtual Temperature

- Gas law for dry air and water vapor

$$e = \rho_v' R_v T$$
$$p_d' = \rho_d' R_d T$$

  e and p_d' are partial pressures.

- From Dalton's Law

$$p = p_d' + e$$

- Recall that

$$\rho = \rho_d' + \rho_v'$$

Virtual Temperature

- Combine these expressions

$$\rho = \rho'_d + \rho'_v$$

$$\rho = \frac{p'_d}{R_d T} + \frac{e}{R_v T}$$

$$\rho = \frac{p - e}{R_d T} + \frac{e}{R_v T}$$

$$\rho = \frac{p}{R_d T}\left[1 - \frac{e}{p}(1 - \epsilon)\right]$$

This can be rewritten as

$$p = \rho R_d T_v$$

where

$$T_v \equiv \frac{T}{1 - \frac{e}{p}(1 - \epsilon)}$$

Virtual Temperature

the virtual temperature is the temperature that dry air would need to attain in order to have the same density as the moist air at the same pressure

- Recall that moist air is less dense than dry air for the same T and p
- Thus, T_v is always greater than T
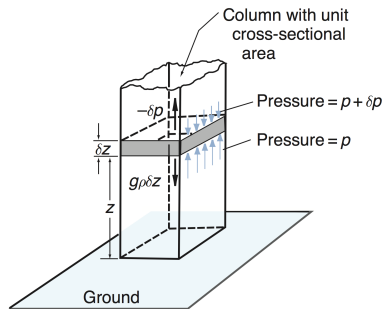- In the case of a very warm and moist air, T_v is only slightly larger than T

Pressure

- Atmospheric pressure at a given height is the due to the weight of the air overlying that height

- Thus, pressure decreases with increasing altitude

- For a thin slice of air, there is upward pressure exerted by pressure gradient force (high to low pressure)

- This is generally close in magnitude to the downward force caused by gravitational acceleration

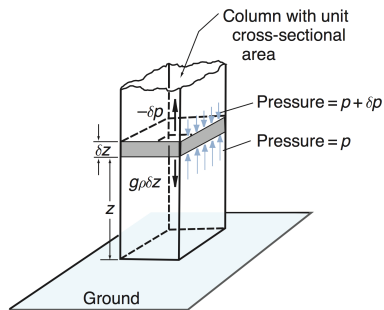- If upward = downward, then we say that the atmosphere is in *hydrostatic balance*

From Wallace and Hobbs (2006)

- Consider column of air w/unit cross-sectional area
- Mass of air between z and $z + \delta z$ is $\rho \delta z$
- Downward force due to gravity is $g\rho \delta z$
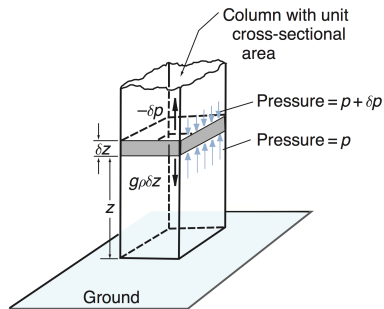
Atmospheric Thermodynamics: Hydrostatic Equation

From Wallace and Hobbs (2006)

- Change in pressure between z and δz is δp

- Since pressure decreases with height, $\delta p < 0$, and upward force on bottom is slight bigger than downward force on top

- Thus, net vertical force due to pressure is $-\delta p$

From Wallace and Hobbs (2006)

- If in hydrostatic balance

$$-\delta p = g\rho\delta z$$

or in limit of $\delta z \rightarrow 0$

$$\frac{\partial p}{\partial z} = -\rho g$$

the so-called **hydrostatic equation**

$$\partial p / \partial z = -\rho g$$

- Above a fixed point on Earth

$$-\int_{p(z)}^{p(\infty)} dp = \int_{z}^{\infty} \rho g \, dz$$

Since $p(\infty) = 0$, we have

$$p(z) = \int_{z}^{\infty} \rho g \, dz$$

- Thus, the pressure at height z is the weight of the air in the vertical column above it
- The ideal sea level pressure (assumes mass of atmosphere distributed evenly) is 1.013×10^{5} Pa $= 1013$ hPa - $1 \ atmsophere$

Geopotential

at a point it is the work required to be done against gravity to raise a mass of 1 kg *from seal level to that point*

- In other words, the geopotential Φ is the gravitational potential per unit mass
- Units $= [\mathrm{J\ kg^{-1}}]$ or $[\mathrm{m^2 s^{-2}}]$

Geopotential

- Force acting on 1 kg at height z is equal to g
- Thus, work required to raise that from z to $z + dz$ is gdz
- Accordingly

$$d\Phi \equiv gdz$$

- So, the geopotential at a given height is

$$\Phi(z) = \int_0^z gdz$$

Geopotential

- We can defined **geopotential height** as

$$Z \equiv \frac{\Phi(z)}{g_0} = \frac{1}{g_0} \int_0^z g \, dz$$

where g_0 is a globally-averaged gravitational acceleration (9.81 m s^{-2})

- Generally in the areas of meteorological importance, $g \approx g_0$ and so $z \approx Z$

Geopotential

Table 3.1 Values of geopotential height (Z) and acceleration due to gravity (g) at 40° latitude for geometric height (z)

z (km)	Z (km)	g (m s^{-2})
0	0	9.81
1	1.00	9.80
10	9.99	9.77
100	98.47	9.50
500	463.6	8.43

Wallace and Hobbs (2006)

Geopotential

- Inconvenient to deal with ρ.

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{pg}{RT} = -\frac{pg}{R_d T_v} \Rightarrow g dz = -R_d T_v \frac{dp}{p}$$

Substitute into geopotential formulation

$$d\Phi = g dz = -R_d T_v \frac{dp}{p}$$

Geopotential

$$d\Phi - R_d T_v \frac{dp}{p}$$

- Let's integrate between two pressure levels p_1 and p_2, with geopotentials Φ_1 and Φ_2

$$\int_{\Phi_1}^{\Phi_2} = -\int_{p_1}^{p_2} R_d T_v \frac{dp}{p} \Rightarrow \Phi_2 - \Phi_1 = -R_d \int_{p_1}^{p_2} T_v \frac{dp}{p}$$

- Divide by g_0 and reverse limits of integration

$$\frac{\Phi_2}{g_0} - \frac{\Phi_1}{g_0} = \frac{R_d}{g_0} \int_{p_2}^{p_1} T_v \frac{dp}{p} \Rightarrow Z_2 - Z_1 = \frac{R_d}{g_0} \int_{p_2}^{p_1} T_v \frac{dp}{p}$$

- $Z_2 - Z_1$ is called the **geopotential thickness** between levels p_2 and p_1