Environmental Fluid Dynamics: Lecture 3

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Overview

A Few More Things About Radiation

Black Body Radiation

From Bergman et al. (2011):

> *A black body absorbs all incident radiation, regardless of wavelength and direction*

- for a prescribed temperature and wavelength, no surface can emit more energy than a blackbody
- Although the radiation emitted by a black body is a function of wavelength and temperature, it is independent of direction (*i.e.*, the blackbody is a diffuse emitter)
- a blackbody is a perfect absorber and emitter, letting it serve as the standard against which the radiative properties of actual surfaces may be compared

Black Body Radiation

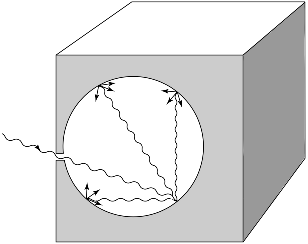


Figure 1.6 A blackbody radiation cavity to illustrate that absorption is complete.

From Liou (2002)

- consider a cavity with a small entrance hole
- most radiant flux entering this hole will be trapped within the cavity
- this occurs regardless of the material and surface properties of the wall

Black Body Radiation

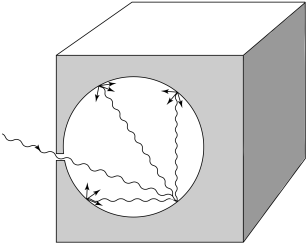


Figure 1.6 A blackbody radiation cavity to illustrate that absorption is complete.

From Liou (2002)

- internal reflections occur until all the fluxes are absorbed by the wall
- chance that any of the entering flux will escape back through the hole is so small that the interior appears dark
- *blackbody* is used for a configuration of material where absorption is complete

Planck's Law

From Bergman et al. (2011):

Planck's Law describes the blackbody spectral emissive power

$$E_{\lambda,T}(\lambda, T) = \frac{C_1}{\lambda^5 \left[\exp(C_2/\lambda T) - 1\right]}$$

where

$$C_1 = 3.742 \times 10^8 \ \mathrm{W\mu m^4 m^{-2}}$$

$$C_2 = 1.439 \times 10^4 \ \mathrm{\mu m\ K}$$

Planck's Law

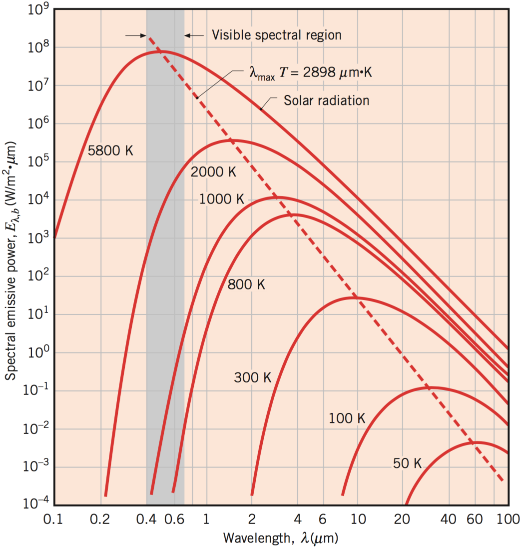
From Bergman et al. (2011):

$$E_{\lambda,T}(\lambda, T) = \frac{C_1}{\lambda^5 \left[\exp(C_2/\lambda T) - 1\right]}$$

- blackbody radiant intensity increases with temperature
- wavelength of the maximum intensity decreases with increasing temperature

Planck's Law

From last class, this is Planck's Law



Bergman et al. (2011)

Stefan-Boltzman Law

- The total hemispherical missive power (the rate that radiation is emitted per unit area at all wavelengths and in all direction) is

$$E = \int_0^\infty E_\lambda(\lambda) d\lambda$$

- Substituting Planck's Law into this equation gives

$$E = \int_0^\infty \frac{C_1}{\lambda^5 \left[\exp(C_2/\lambda T) - 1\right]} d\lambda$$

- Performing the integration yields the Stefan-Boltzman Law

Stefan-Boltzman Law

Gives the amount of radiation emitted in all directions and over all wavelengths simply from knowledge of the temperature of the blackbody

$$E_b = \sigma T^4$$

where

$$\sigma = 5.670 \times 10^8 \ \mathrm{Wm^{-2}\ K^4}$$

In practice, the Law is usually written in terms of an object's emissivity ϵ. For the surface of the Earth:

$$R_{L\uparrow} = -\epsilon \sigma T_s^4$$

Note: minus sign because we are talking about upward directed radiation

Stefan-Boltzman Law

$$E_b = \sigma T^4$$

where

$$\sigma = 5.670 \times 10^8 \text{ W m}^{-2} \text{ K}^4$$

- the flux density emitted by a blackbody is proportional to the 4^{th} power of the absolute temperature
- the S-B law is fundamental to the analysis of broadband infrared radiative transfer

Wein's Displacement Law

It is obtained by differentiating Planck's Law with respect to wavelength and setting the result to zero

$$\frac{\partial E_{\lambda,T}}{\partial \lambda} = 0$$

The results gives Wein's Displacement Law

The wavelength of the maximum intensity of blackbody radiation is inversely proportional to the temperature

$$\lambda_m = \frac{a}{T}$$

where $a = 2898 \text{ m K}$

Wein's Displacement Law

The wavelength of the maximum intensity of blackbody radiation is inversely proportional to the temperature

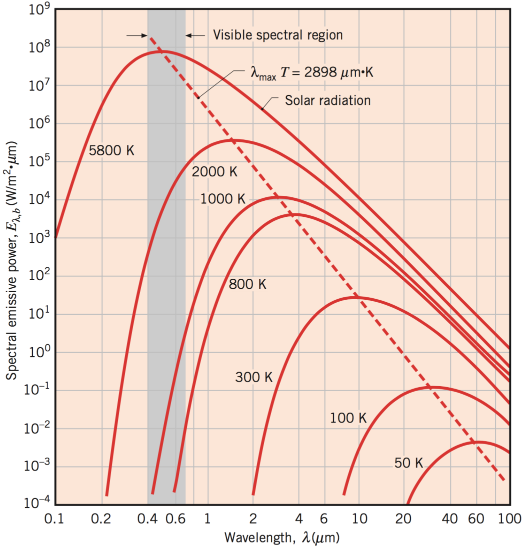
$$\lambda_m = \frac{a}{T}$$

where $a = 2898 \text{ m K}$

- one can determine the temperature of a blackbody from the measurement of the maximum intensity
- this dependence of maximum intensity on temperature is clearly seen in the previous figure when discussing Planck's Law

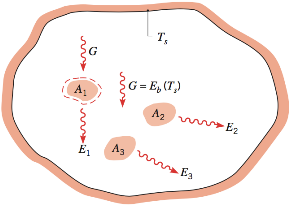
Wein's Displacement Law

Note the dependence of peak intensity location on temperature



Bergman et al. (2011)

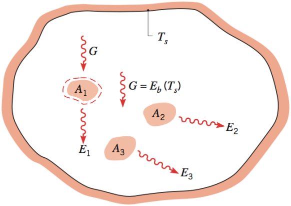
Kirchhoff's Law



From Bergman et al. (2011)

- consider a perfectly insulated enclosure having black walls
- assume the system has reached thermodynamic equilibrium
- radiation emitted by the system to the walls is absorbed and the same amount of radiation absorbed by the walls is also emitted

Kirchhoff's Law

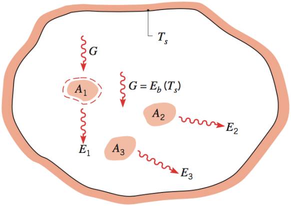


From Bergman et al. (2011)

- the emissivity of a given wavelength, ϵ_λ ($\frac{\text{emitting intensity}}{\text{Planck function}}$), of a medium is equal to the absorptivity, A_λ ($\frac{\text{absorbed intensity}}{\text{Planck function}}$), of that medium under thermodynamic equilibrium

$$A_\lambda = \epsilon_\lambda = 1$$

Kirchhoff's Law



From Bergman et al. (2011)

- It can be shown that each of the confined bodies in the enclosure must adhere to:

$$\frac{E_1(T_s)}{A_{\lambda 1}} = \frac{E_2(T_s)}{A_{\lambda 2}} = \ldots = E_b(T_s)$$

- since $A_\lambda \leq 1$, $E(T_s) \leq E_b(T_s)$
- no real surface can have an emissive power exceeding that of a black surface at the same temperature
- thus, a blackbody is confirmed to be a perfect emitter

Introduction to Heat Transfer in Soils and Other Materials

Surface/Skin Temperature

T_s - The temperature at the air-soil interface.

An "ideal" surface has uniform T_s and varies in time in response to energy fluxes at the surface. Inhomogeneous surfaces are likely to have spatially varying surface temperatures.

T_s depends on

- radiation balance
- surface exchange processes (*e.g.*, turbulence)
- vegetative cover
- thermal properties of the subsurface

Surface/Skin Temperature

Data from MATERHORN field campaign for a playa site at the U.S. Army Dugway Proving Grounds. Note the large surface temperature heterogeneities

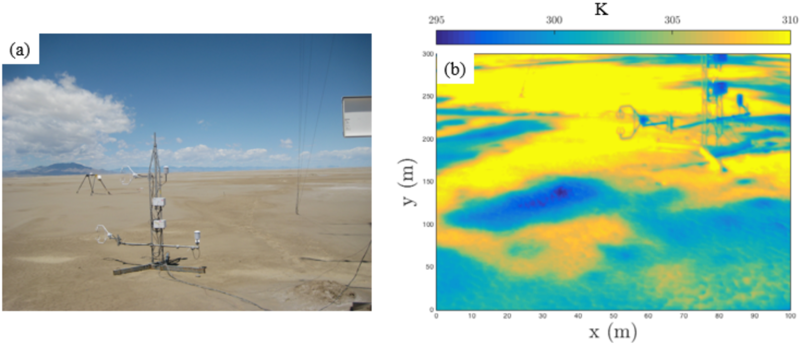


figure courtesy Travis Morrison

Surface/Skin Temperature

T_s - The temperature at the air-soil interface.

T_s is difficult to measure

- very large temperature gradients near the surface both in the air and soil ($10 - 20\ \mathrm{K\ mm^{-1}}$ possible in air above bare heated soil!)
- usually computed by extrapolating air/soil temperatures
- radiometer - uses $R_{L\uparrow} \sim -\epsilon\sigma T_s^4$ (Stefan-Boltzman Law)

Diurnal Range

- in a dry desert: $\sim 40\text{-}50\ ^\circ\mathrm{C}$
- Surface and subsurface moisture moderate the range
  - increased evaporation from the surface
  - increased heat capacity (c) and conductivity of the soil (k)
  - wet soils may dry changing the temperature response
- Vegetation moderates diurnal range moderate the range
  - intercepts incoming solar radiation $\rightarrow$ lower surface temps during the day
  - intercepts outgoing longwave radiation
  - enhanced latent heat flux due to evapotranspiration (ET)
  - increased turbulence

Diurnal Soil and Temperature

- On a clear day, the max surface temperature is generally reached about 1-2 hours after peak incoming solar radiation (insolation; solar noon)
- The min surface temperature occurs in the early morning hours
- On a larger scale, this is why the maximum annual temperatures (northern hemisphere) are not reached in June even though this time has peak insolation

Diurnal Soil and Temperature

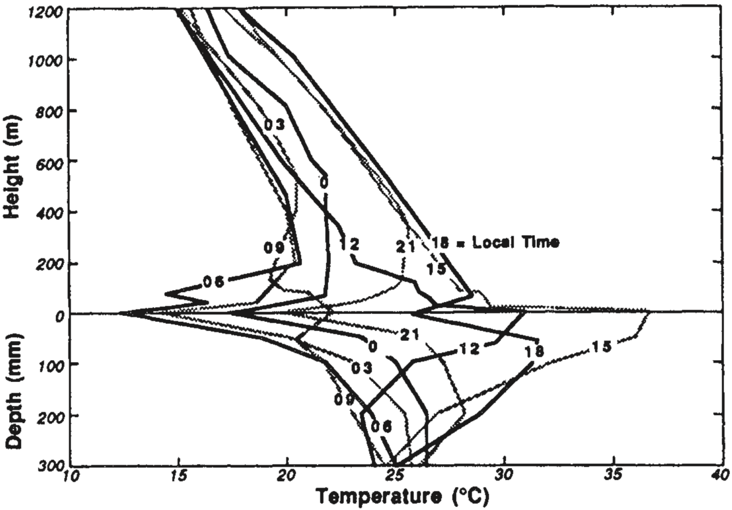


Fig. 7.17 Three day average of temperature profiles at indicated hours for the Koorin field program, days 7-9. Note the scale difference between height and depth. (After Lettau, personal communication).

Stull (1988)

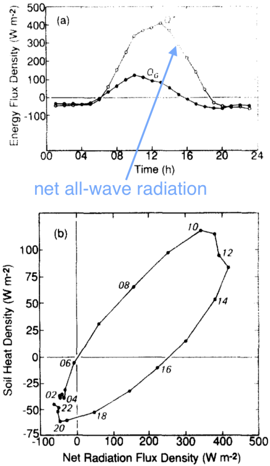


Fig. 1. (a) Time series and (b) hysteresis loop relations between soil heat flux and net all-wave radiation for short grass near St Louis, MO calculated from the data of Doll *et al.* (1985) for a single day. Best fit statistics give the equation: $Q_G = 0.32Q^* + 0.54(\partial Q^*/\partial t) - 27.4$.

hysteresis

the phenomenon in which the value of a physical property lags behind changes in the effect causing it

the dependence of the state of a system on its history

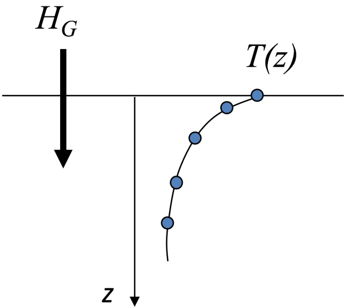
$$\Delta H_s = \int \frac{\partial(\rho C T)}{\partial t} dz$$

Sub-surface Soil Temperature

- much easier to measure - thermocouple
- amplitude of the temperature fluctuations decrease exponentially with depth
- depends on:
  - latitude
  - time of year
  - net radiation
  - soil texture (porosity) and moisture content
  - ground cover
  - surface weather conditions

Thermal Properties of Soil

- Specific Heat - c ($\mathrm{J\ kg^{-1}\ K^{-1}}$) - the amount of heat absorbed by a material to raise the temperature of a unit mass of material by $1\mathrm{K}$
- Thermal Conductivity - k ($\mathrm{W\ m^{-1}\ K^{-1}}$) - material property; the ability of a material to conduct heat
- Thermal Diffusivity - α_h ($\mathrm{m^2\ s^{-1}}$) - ratio of thermal conductivity to heat capacity



1D Thermal Conduction

$$H_G = -k\frac{\partial T}{\partial z}$$ Fourier's conduction law

$$\alpha_h = \underbrace{\frac{k}{\boxed{\rho c}} = \frac{k}{\boxed{C}}}_{\text{soil heat capacity}}$$

$$\frac{\partial(\rho c T)}{\partial t} = -\frac{\partial H_G}{\partial z}$$

$$\frac{\partial(\rho c T)}{\partial t} = k\frac{\partial^2 T}{\partial z^2}$$

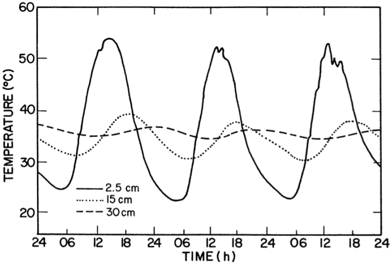
$$\boxed{\frac{\partial T}{\partial t} = \alpha_h\frac{\partial^2 T}{\partial z^2}}$$

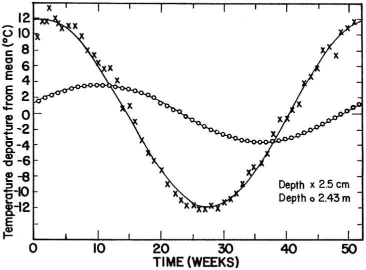
Solutions

$$\frac{\partial T}{\partial t} = \alpha_h \frac{\partial^2 T}{\partial z^2}$$

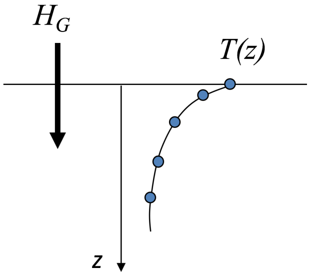
- analytical - multiple methods
- numerical - *e.g.*, finite difference
- force-restore - 2 layer slab model (see Stull Ch.7, Blackadar 1976)

Diurnal and Annual Soil Temperature Temporal Variability





diurnal wave (left), **annual wave** (right)



1D Thermal Conduction

$$\frac{\partial(CT)}{\partial t} = -\frac{\partial H}{\partial z}$$

$$H(z=0) - H(z=D) = \int_{z=0}^{z=D} \frac{\partial(CT)}{\partial t} dz$$

$$\underbrace{H_G}_{\text{ground heat flux}} = \underbrace{H_D}_{\text{reference depth}} + \underbrace{\int_{z=0}^{z=D} \frac{\partial(CT)}{\partial t} dz}_{\text{storage}}$$

Soil Heat Transfer: Governing Parameters

- thermal conductivity - k
- heat cpacity - C_s
- thermal diffusivity - α (sometimes κ)
- thermal admittance - μ

Soil Heat Transfer: Governing Parameters

thermal conductivity k ($\mathrm{W\ m^{-1}\ K^{-1}}$)

- the ability of a material to conduct heat
- depends on:
  - soil particles
  - soil porosity
  - moisture content

Soil Heat Transfer: Governing Parameters

heat capacity $C_s = \rho c$ $(\mathrm{J\ m^{-3}\ K^{-1}})$

- c - specific heat of the soil $(\mathrm{J\ kg^{-1}\ K^{-1}})$
- relates to the ability of a material to store heat
- the amount of heat (J) necessary to increase a unit volume $(\mathrm{m^3})$ of a substance by $1\mathrm{K}$
- water $(\sim 5\ \mathrm{J\ m^{-3}\ K^{-1}})$ has a very high heat capacity, air is quite low
- Depends on:
  - porosity
  - mineral content
  - organic content
  - air, etc.

Soil Heat Transfer: Governing Parameters

thermal diffusivity $\alpha = k/C_s$ $(\mathrm{m}^2\ \mathrm{s}^{-1})$

- Controls the speed at which temperature waves move through the soil and the depth of thermal influence of an active surface
- In other words, consider it to be the rate of transfer of heat in a material from the "hot side" to the "cold side"
- water $(\sim 1.43 \times 10^{-7}\ \mathrm{m}^2\ \mathrm{s}^{-1})$
- air $(\sim 1.9 \times 10^{-5}\ \mathrm{m}^2\ \mathrm{s}^{-1})$

Soil Heat Transfer: Governing Parameters

thermal admittance $\mu = \sqrt{kC_s}$ $(\mathrm{J\ m^{-2}\ s^{-1/2}\ K^{-1}})$

- a surface property - not a soil property
- the ability of a surface to accept or release heat
- high μ - metals
- low μ - wood
- high μ materials feel cooler to the touch even though they have the same surface temperature

Soil Heat Transfer: Governing Parameters Typical Values

Table 2.1 Thermal properties of natural materials

Material	Remarks	ρ Density (kg m^{-3} × 10^3)	c Specific heat (J kg^{-1} K^{-1} × 10^3)	C Heat capacity (J m^{-3} K^{-1} × 10^6)	k Thermal conductivity (W m^{-1} K^{-1})	κ Thermal diffusivity (m^2 s^{-1} × 10^{-6})	μ Thermal admittance (J m^{-2} s$^{-1/2}$ K^{-1})
Sandy soil (40% pore space)	Dry	1·60	0·80	1·28	0·30	0·24	620
	Saturated	2·00	1·48	2·96	2·20	0·74	2550
Clay soil (40% pore space)	Dry	1·60	0·89	1·42	0·25	0·18	600
	Saturated	2·00	1·55	3·10	1·58	0·51	2210
Peat soil (80% pore space)	Dry	0·30	1·92	0·58	0·06	0·10	190
	Saturated	1·10	3·65	4·02	0·50	0·12	1420
Snow	Fresh	0·10	2·09	0·21	0·08	0·10	130
	Old	0·48	2·09	0·84	0·42	0·40	595
Ice	0°C, pure	0·92	2·10	1·93	2·24	1·16	2080
Water*	4°C, still	1·00	4·18	4·18	0·57	0·14	1545
Air*	10°C, still	0·0012	1·01	0·0012	0·025	21·50	5
	Turbulent	0·0012	1·01	0·0012	~125	~10 × 10^6	390

Oke (1987)

Effect of Soil Moisture on Thermal Properties

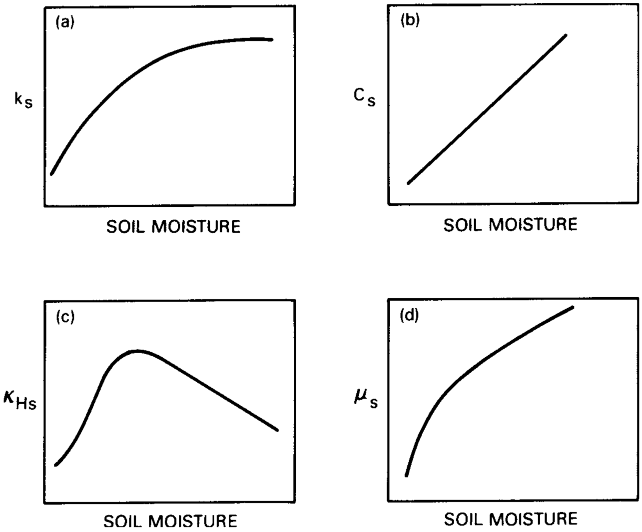


Figure 2.5 Relationship between soil moisture content: (a) thermal conductivity, (b) heat capacity, (c) thermal diffusivity and (d) thermal admittance for most soils.

Variation of Fluxes in Urban Areas

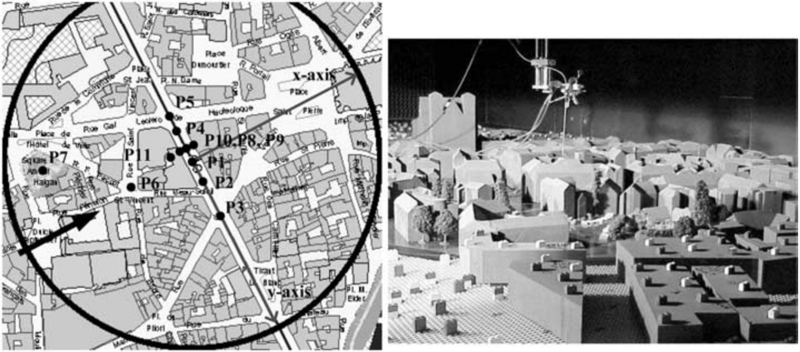


Figure 2. Area in the centre of Nantes reconstructed in the wind-tunnel model (circle) with the location of profiles (left) and photograph of the model installed in the boundary-layer wind tunnel (right).

From Kastner-Klein, P., and Rotach, M. W. (2004). Mean Flow and Turbulence Characteristics in an Urban Roughness Sublayer. Boundary-Layer Meteorology, 111(1), 55–84.

Variation of Fluxes in Urban Areas

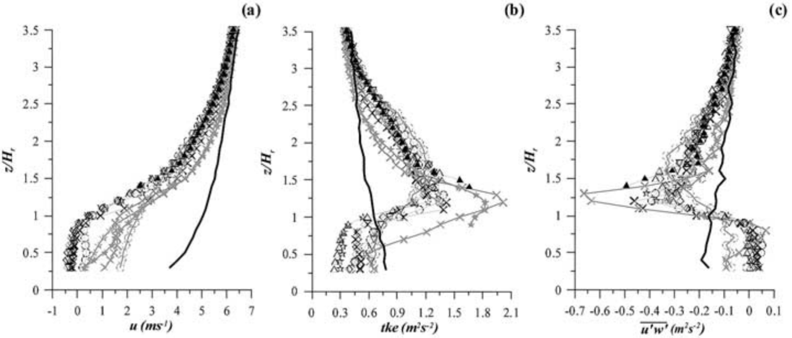


Figure 5. Measured profiles of (a) the mean *u* component, (b) turbulence kinetic energy and (c) turbulent shear stress of the Nantes wind-tunnel study (symbols according to Table II).

From Kastner-Klein, P., and Rotach, M. W. (2004). Mean Flow and Turbulence Characteristics in an Urban Roughness Sublayer. Boundary-Layer Meteorology, 111(1), 55–84.

Urban Areas

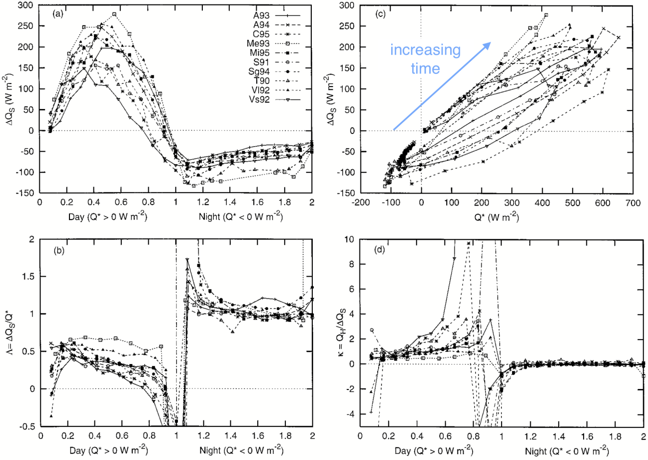


FIG. 1. Mean diurnal patterns of observed (a) ΔQ_S (W m^{-2}), (b) $\Delta Q_S/Q^*$, (c) ΔQ_S vs Q^*, and (d) $Q_H/\Delta Q_S$ for each of the datasets (see details in Table 1).

From Grimmond and Oke (1999)

More energy is transferred to the “urban fabric” in the morning - asymmetry

Urban Areas

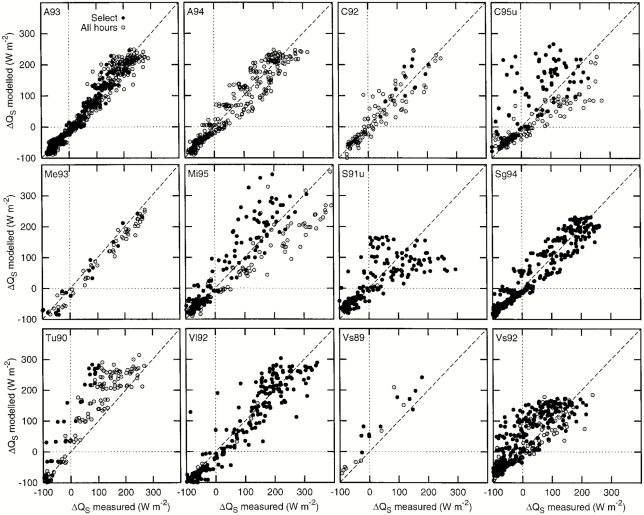


FIG. 6. Scatterplots of two-hourly measured vs modeled storage heat flux (W m^{-2}) (see text for explanation). Solid circles are hours from selected wind directions (Table 4); open circles all other hours of observation. Statistics of goodness of fit reported in Table 6.

From Grimmond and Oke (1999)