LES of Turbulent Flows: Lecture 24

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Overview

1. Special Topic: WALE Model

Special Topic: WALE Model

- Recall that many of the LES SGS models we have covered are variants of the eddy-viscosity assumption

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = 2\nu_T \widetilde{S}_{ij}$$

where ν_T is the eddy-viscosity term that must be modeled and

$$\widetilde{S}_{ij} = \left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i} \right)$$

is the strain rate (deformation) tensor of the resolved flow field

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- In the case of the Smagorinsky model, we showed that

$$\nu_T = (C_S \Delta)^2 |\widetilde{S}|$$

where $\Delta = (\Delta_x \Delta_y \Delta_z)^{\frac{1}{3}}$ is the effective grid scale, C_S is the Smagorinsky coefficient, and

$$|\widetilde{S}| = \sqrt{2 \widetilde{S}_{ij} \widetilde{S}_{ij}}$$

is the magnitude of the filtered strain rate tensor

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- Finally, we showed that if:
  1. we used a spectral cutoff filter,
  2. assume the cutoff wavenumber is in the inertial subrange of a Kolmogorov-type spectrum,
  3. and equate viscous dissipation with the ensemble-average SGS dissipation ($\epsilon = \langle \Pi \rangle$), then

$$C_S = \frac{1}{\pi}\left(\frac{2}{3C_k}\right)^{\frac{3}{4}}$$

where if we assume $C_K \approx 1.4$ we get

$$\boxed{C_S \simeq 0.18}$$

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- Nicoud and Ducros (1999) challenged the underlying assumption of the Smagorinsky model – that the local strain rate of the flow defines the velocity scale at the filter width
- ND99 also investigated how such a model should behave near the lower boundary (wall)
- Additionally, the topic of complex geometry and numerical methods were discussed

Special Topic: WALE Model

- Recall that ν_T acts to represent the transfer of energy from resolved to subgrid scales through subgrid dissipation ($\propto \nu_T$)
- Accordingly, using the Smagorinsky-style model, the subgrid dissipation is described by the strain rate of the smallest resolved scales
- However, Wray and Hunt (1989) showed from DNS of isotropic turbulence that energy is actually concentrated around zones of vorticity and strain
- Thus, using only the strain rate is inadequate

Special Topic: WALE Model

- ND99 suggested a better SGS model would account for both strain rate and rotational rate
- One example is an eddy-viscosity model based on the structure function

$$\nu_T = \beta C_k^{-3/2} \Delta \sqrt{\widetilde{F_2}}$$

where $\widetilde{F_2}$ is the second-order velocity structure function of the filtered field

$$\widetilde{F_2}(\vec{x}, \Delta) = \langle |\widetilde{\vec{u}}(\vec{x} + \vec{r}, t) - \widetilde{\vec{u}}(\vec{x}, t)|^2 \rangle$$

and β is a constant (Lesieur and Metais 1999 suggested $\beta = 0.105$ for isotropic turbulence)

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- Another issue beyond the physical description of the model is how it behaves near the wall
- The Smagorinsky model will give non-zero values of ν_T provided gradients exist – however in the reality turbulent fluctuations are damped near the wall so that ν_T should be zero
- One way to correct this in the past was the Van Driest exponential damping function

$$1 - exp(-y^+/A^+) \qquad \text{where } A^+ = 25$$

- Recall from Lecture 14 that Deardorff also implemented an enhancement of dissipation near the wall to prevent unnatural build-up of energy

Special Topic: WALE Model

- While these damping methods improve results, they are *ad hoc* and are difficult to apply to complex geometries
- ND99 notes that there are ways to try and force near-zero ν_T, which includes limiting the distance over which $\widetilde{F_2}$ is computed, or computing C_S dynamically (as in Germano)
- Recall, however, that the dynamic procedure may result in a large fraction of negative C_S values, which can lead to numerical instability
- While spatial averaging can alleviate this problem, the required size of the stencil is not really known *a priori*, it is *ad hoc*, and it is limited to simple flow geometries

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- The general form of these models can be given by

$$\nu_T = C_m \Delta^2 \widetilde{OP}(\vec{x}, t)$$

  where C_m is the model coefficient and $\widetilde{OP}$ is an operator defined from the resolved scales
- ND99 proposed a new operator with the following properties
  - invariant to coordinate rotation or translation
  - easily assessed on any computational grid
  - function of both strain and rotation rates
  - goes naturally to 0 at the wall
- Called the Wall-Adapting Local Eddy-viscosity (WALE) model

Special Topic: WALE Model

- $\widetilde{OP}$ must be based on invariants of a tensor τ_{ij}, which represents turbulence
- Velocity gradient tensor $(\widetilde{g}_{ij} = \partial \widetilde{u}_i / \partial x_j)$ is a good candidate
- Smagorinsky was based on second invariant of the symmetric part of $\widetilde{S}_{ij}$ of this tensor (see Lecture 7)
- Two drawbacks with Smagorinsky approach:
  1.) only considers strain rate of structures, not rotational rate
  2.) leads to unphysical $\nu_T = \mathcal{O}(1)$ at the surface

Special Topic: WALE Model

- ND99 constructed a new proposed (and hopefully better) operator by considering the traceless symmetric part of the square of the velocity gradient tensor

$$S_{ij}^d = \frac{1}{2}\left(\widetilde{g}_{ij}^2 + \widetilde{g}_{ji}^2\right) - \frac{1}{3}\delta_{ij}\widetilde{g}_{kk}^2$$

  where $\widetilde{g}_{ij}^2 = \widetilde{g}_{ik}\widetilde{g}_{kj}$

- Note: the antisymmetric part of $\overline{g}$ is given by

$$\widetilde{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i}\right)$$

Special Topic: WALE Model

- After some math (see ND99 on Canvas or website), the WALE model is given by

$$\nu_T = (C_w \Delta)^2 \frac{\left(S^d_{ij} S^d_{ij}\right)^{3/2}}{\left(\widetilde{S}_{ij}\widetilde{S}_{ij}\right)^{5/2} + \left(S^d_{ij} S^d_{ij}\right)^{5/4}}$$

where

$$C_w^2 = C_S^2 \frac{\left\langle \sqrt{2}\left(\widetilde{S}_{ij}\widetilde{S}_{ij}\right)^{3/2} \right\rangle}{\left\langle \widetilde{S}_{ij}\widetilde{S}_{ij}\left(S^d_{ij} S^d_{ij}\right)^{3/2}\left(\left(\widetilde{S}_{ij}\widetilde{S}_{ij}\right)^{5/2} + \left(S^d_{ij} S^d_{ij}\right)^{5/4}\right)^{-1} \right\rangle}$$

For $C_S \approx 0.18$, expect $0.55 \leq C_w \leq 0.60$

Special Topic: WALE Model

- The WALE model formulation accounts for rotational rate, naturally goes to 0 at the wall without the need for any *ad hoc* methods, and can be generalized for any grid and complex geometries

Special Topic: WALE Model

- One first test was to see how well the WALE model produced spectra for decaying isotropic turbulence

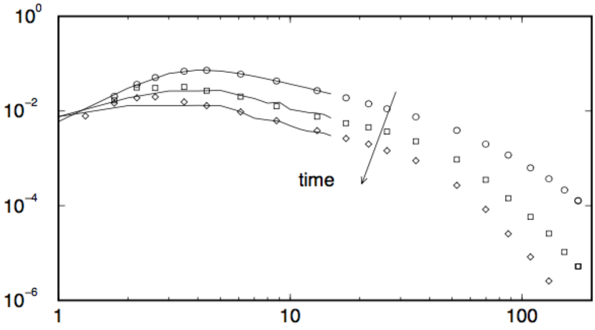

Figure 1: Time evolution of energy spectra for freely decaying isotropic turbulence with the WALE model. The grid contains 32^3 points. Symbols are experimental mesurements. Times are 42, 98 and 171 M/U_o

Special Topic: WALE Model

- In the next test ND99 examined the WALE model in a turbulent pipe flow using a hybrid grid

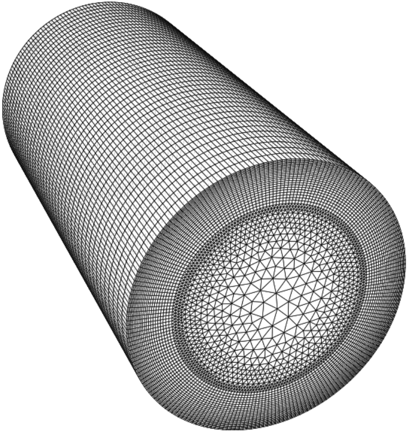

Figure 2: The hybrid grid used for the LES of a turbulent pipe flow.

Special Topic: WALE Model

- Results from turbulent pipe flow tests

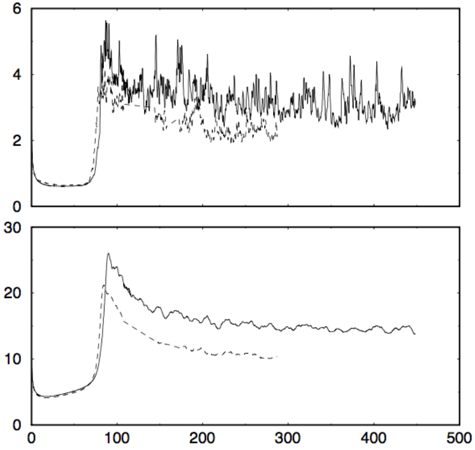

Figure 3: Time evolution of kinetic energy (bottom) and of maximum of vorticity (top) with both the filtered-Smagorinsky (– – –) and the WALE (——-) models. Time unit is R/U_b

Special Topic: WALE Model

- Results from turbulent pipe flow tests

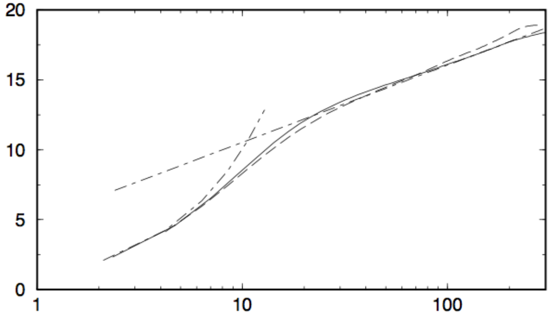

Figure 5: Mean velocity profile vs. the distance to the wall (semi-log coordinates). Comparison between the filtered-Smagorinsky model (– – –) and the WALE formulation (——). The law-of-the-wall is denoted by the dot-dashed line.

Special Topic: WALE Model

- Results from turbulent pipe flow tests

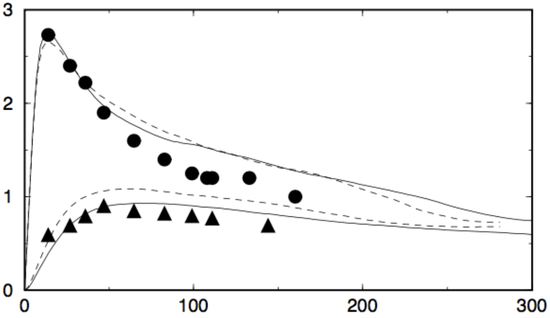

Figure 6: Root-mean-square streamwise velocity and normal velocity vs. the distance to the wall. Comparison between the filtered-Smagorinsky model (– – –) and the WALE formulation (——). The experimental data from Eggels et al. for $R_b = 5450$ is denoted by symbol.