LES of Turbulent Flows: Lecture 22

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Overview

1. Surface/Wall Boundary Conditions
   - Requirements to Resolve the Wall
   - Approximate Wall-Boundary Conditions
   - Accounting for Flow Average Flow Structures

2. Local and Higher-order RANS Approximations

Surface/Wall Boundary Conditions

- In many flows of interest, a solid wall (or surface) is present in some way
- It can be very costly to fully resolve the effects of the wall and implement “natural” no-slip BCs
- Chapman (1979) performed the first analysis of grid-resolution requirements for LES of wall-bounded flows

Surface/Wall Boundary Conditions

We can divide the flow into 2 regions:

- **Outer layer**: viscosity isn't as important and grid resolution requirements are more or less (not including SGS model errors) independent of Re
- **Inner layer**: near wall region where viscosity plays an important role

Surface/Wall Boundary Conditions

Inner layer:

- Structures ("eddies") in the inner-layer are approximately constant when non-dimensionalized with viscous length scales
- To resolve these motions we need grid spacing of

$$\Delta x^+ \sim 100 \quad (x^+ = x_i u_\tau / \nu)$$
$$\Delta z^+ \sim 20$$

where $u_\tau = \sqrt{\dfrac{\tau_w}{\rho}}$ is the friction velocity

Requirements to Resolve the Wall

- Using these Δx^+ and Δz^+ scales, we can show that

$$N_x \times N_y \times N_z \propto \mathrm{Re}_L^{1.8}$$

  where Re_L is the integral scale Reynolds number – that is the Reynolds number that is based on the integral length scale of turbulence
- The integral length scale is the characteristic length scale of the larger eddies in a turbulent flow
- In order to resolve the viscous sublayer (to enforce the use of the no-slip condition), the number of required grid points scales as $\mathrm{Re}_L^{1.8}$
- Conversely, Chapman (1979) showed that the number of grid points required to resolve the outer layer scales as $\mathrm{Re}_L^{0.4}$

Requirements to Resolve the Wall

- For a BL with $\mathrm{Re}_L = 10^6$ (moderate-low Re), **99% of our grid points** must be in the near wall region
- This region is only **10% of the entire boundary layer!**

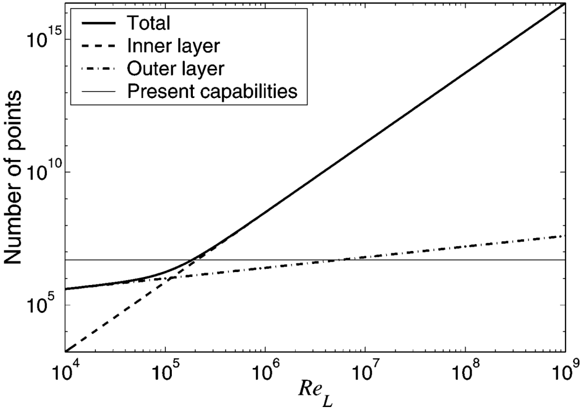


Figure 1 Number of grid points required to resolve a boundary layer. The "Present capabilities" line represents calculations performed on a Pentium III 933MHz workstation with 1Gbyte of memory.

Approximate Wall-Boundary Conditions

- How do we handle this problem for high-Re boundary layers?
- Answer: with **approximate wall-boundary conditions**
  - We pick our first grid-point to be sufficiently far from the wall so it lies in the outer layer
  - This has the **potential to make our simulations only weakly dependent on Re and grid resolution** (if we don't consider model errors!)
  - The **goal is to create a model** that calculates the **wall shear stress as a function of the resolved velocity** at the lowest grid level
  - **All of the dynamics of the inner layer** must be accounted for with the wall model

Approximate Wall-Boundary Conditions

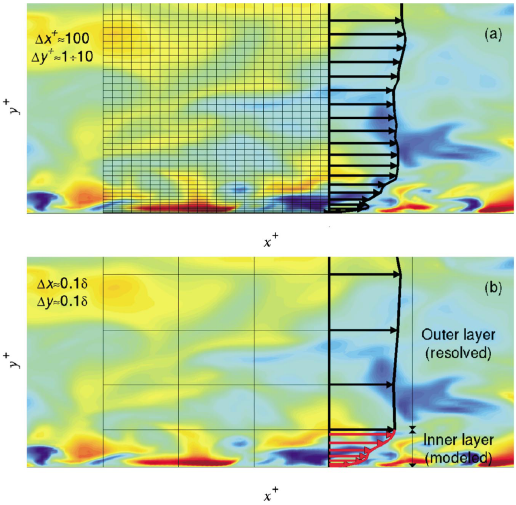


Figure 2 Sketch illustrating the wall-layer modeling philosophy. (*a*) Inner layer resolved. (*b*) Inner layer modeled.

From Piomelli and Balaras (2002)

Approximate Wall-Boundary Conditions

Typical high-Re wall models

- Many wall models use RANS-like approximations
- In high-Re BLs, the most common models are 0^{th}-order RANS (*i.e.* similarity theory)
- $\tilde{u}_i$ and τ_w are assumed to be related by the well known log-law
- For a rough-wall:

$$U(z) = \frac{u_\tau}{\kappa}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_M\left(\frac{z}{L}\right)\right]$$

where $U(z)$ is the mean velocity, $u_\tau = \sqrt{-\tau_w}$ is friction velocity, z is the height of the first model level, z_o is the surface roughness, and Ψ_M is the stability correction function

Approximate Wall-Boundary Conditions

Typical high-Re wall models

- Schumann (1975) introduced the
  of this class of models where:

$$\tau_{i3,w}(x,y,t) = \langle \tau_w \rangle \frac{\tilde{u}_i(\vec{x},t)}{U(z)} \quad \text{for } i = 1,2(x,y)$$

- $\langle \tau_w \rangle$ was calculated from the mean pressure gradient

Approximate Wall-Boundary Conditions

Typical high-Re wall models

- Grötzbach (1987) modified this by using the log-law to calculate the average shear stress resulting in the flowing model

$$\tau_{i3,w}(x,y,t) = -\left[\frac{U(z)\kappa}{\ln(z/z_o) - \Psi_M}\right]\left[\frac{\tilde{u}_i(\vec{x},t)\kappa}{\ln(z/z_o) - \Psi_M}\right]$$

- This model has the advantage over Schumann's because it allows the total mass flux to change in time during a simulation
- Both models assume that $\tau_w \sim \tilde{u}_i$

Accounting for Flow Average Flow Structures

- Piomelli et al. (1989) altered the models of Schumann and Grötzbach (SG) in an attempt to account for the structure of the flow field
- Experimental and numerical studies have demonstrated that coherent structures exist in the BL and that they are inclined at oblique angles to the wall (*e.g.* Brown and Thomas 1977)

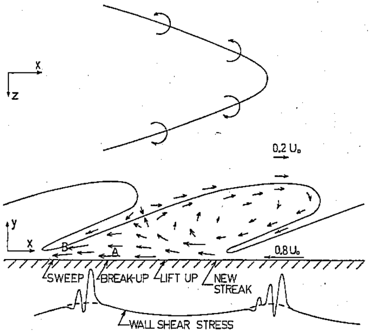
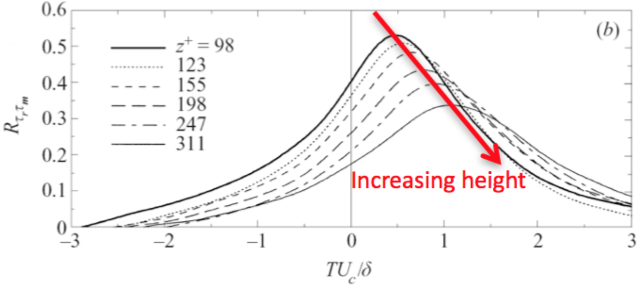


FIG. 12. Proposed flow pattern that might be seen by an observer moving at a speed of 0.8 U_0, and the associated wall shear stress distribution.

Accounting for Flow Average Flow Structures

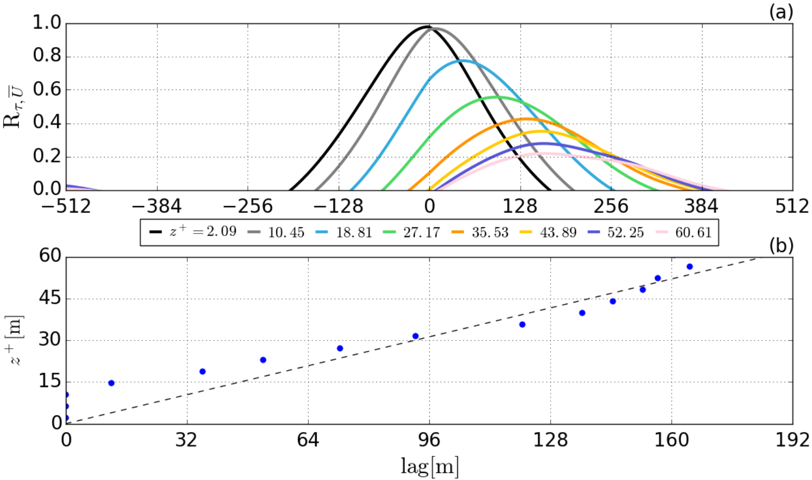
- The inclination of these structures can be measured by looking at the correlation between shear stress and velocity in a BL
- With the average inclination given by the lag to max correlation with height



From Marusic et al (2001)

Accounting for Flow Average Flow Structures

- Another example taken from an idealized LLJ simulation



Accounting for Flow Average Flow Structures

- Piomelli et al. (1989) took this into account by shifting the SG model downstream

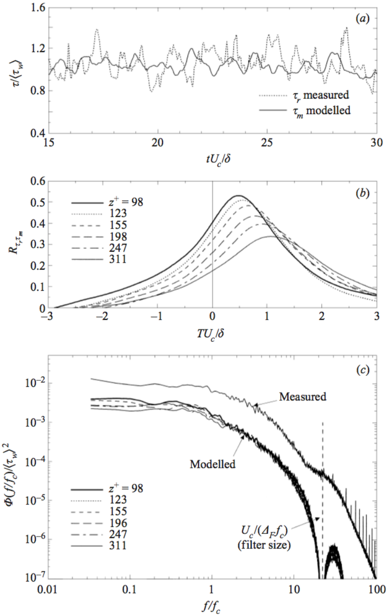
$$\tau_{i3,w}(x,y,t) = \langle \tau_w \rangle \frac{\tilde{u}_i(x+\delta_d, y, z, t)}{U(Z)}$$

where $\delta = z \ \cot(\gamma)$ is the displacement and $\gamma \approx 13^\circ$ for high-Re flows

Approximate Wall-Boundary Conditions

a priori analysis

- Analysis of hotwire data from Marusic et al (2001)
- Found low correlation between SG model and measured data
- Figures show: time series of SG model vs. data (a), 2pt correlations from the SGS model (b) and shear stress spectra from SG model (c) from Marusic et al (2001)



Approximate Wall-Boundary Conditions

a priori analysis

- Based on their analysis, Marusic et al (2001) proposed a new model

$$\tau_{i3,w}(x,y,t) = \langle \tau_w \rangle - \alpha u_\tau \left[\tilde{u}_i(x+\Delta, y, z, t) - \langle \tilde{u}_i(x+\Delta, y, z, t) \rangle \right]$$

- Basic motivation: low frequency filtered velocity spectra will collapse under outer-flow scaling and that the filtered shear stress spectra should follow the filtered velocity spectra
- Based on this, α should be a constant under a variety of conditions

Approximate Wall-Boundary Conditions

a priori analysis

- Following Stoll and Porté-Agel (2006) we can compare this to the SG model

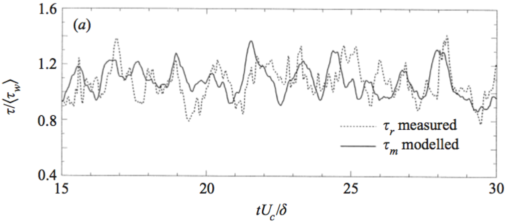
$$\begin{aligned}\tau_{i3,w}(x,y,t) &= \langle\tau_w\rangle \frac{\tilde{u}_i(x+\Delta,y,z,t)}{U_i(z)} \\ &= \langle\tau_w\rangle + \frac{\langle\tau_w\rangle}{U_i(z)}\left[\tilde{u}_i(x+\Delta,y,z,t) - U_i(z)\right] \\ &= \langle\tau_w\rangle - \alpha_{eq} u_\tau \left[\tilde{u}_i(x+\Delta,y,z,t) - U_i(z)\right]\end{aligned}$$

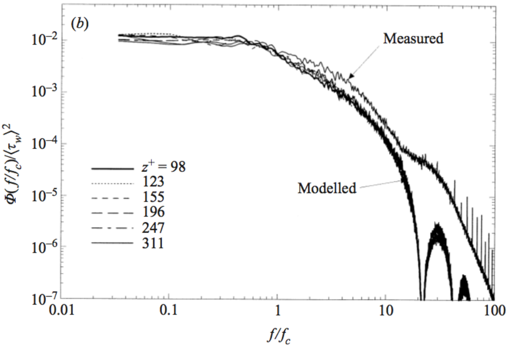
where

$$\alpha_{eq} = \frac{\langle\tau_w\rangle}{u_\tau U_i(z)} = \frac{\kappa}{\ln(z/z_o)}$$

Approximate Wall-Boundary Conditions

a priori analysis





Local and Higher-order RANS Approximations

The local log-law for ABL flows

- In the ABL or general flows where no directions of homogeneity exist for determining $\langle \tau_w \rangle$, the log-law is often used directly to calculate the local shear stress by

$$\tau_{i3,w}(x,y,t) = -\left[\frac{\tilde{u}_r(\vec{x},t)\kappa}{\ln(z/z_o) - \Psi_M}\right]^2 \left[\frac{\tilde{u}_i(\vec{x},t)}{\tilde{u}_r(\vec{x},t)}\right]$$

  where

$$\tilde{u}_r = \sqrt{\tilde{u}_x^2 + \tilde{u}_y^2}$$

- The formulation assumes $\tau_w \sim \tilde{u}_i^2$ and does not preserve $\langle \tau_w \rangle$

2-layer models (higher-order RANS):

- Balaras et al., (AIAA, 1996) used a higher order RANS closure based on the thin-BL equations

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_i}(\tilde{u}_n \tilde{u}_i) = -\frac{\tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_n}\left[(\nu + \nu_T)\frac{\partial \tilde{u}_i}{\partial x_n}\right]$$

where $i = 1, 2$, u_n is the wall normal component found from continuity and ν_T is an eddy-viscosity parameterized with an algebraic model. The equations are solved to the wall.

Local and Higher-order RANS Approximations

2-layer models (higher-order RANS):

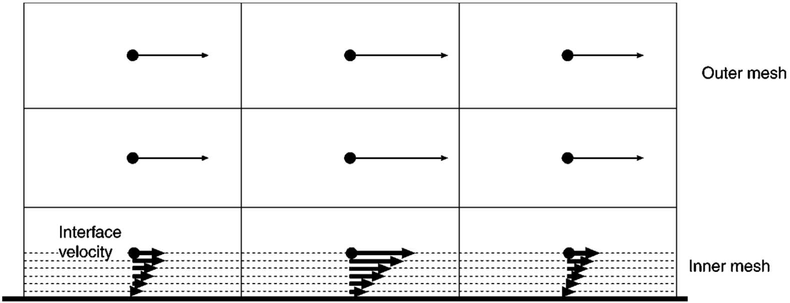


Figure 4 Inner-and outer-layer grids for the two-layer model.

From Piomelli and Balaras (2002)

Even more variations

The filtered local log-law for ABL flows:

- Bou-Zeid et al. proposed to use the filtered velocity to find the surface stress

$$\tau_{i3,w}(x,y,t) = \left[\frac{\bar{\tilde{u}}_i(x+\Delta,y,z,t)\kappa}{\log(z/z_o)}\right]^2 \frac{\tilde{u}_i(x+\Delta,y,z,t)}{\bar{\tilde{u}}_i(x+\Delta,y,z,t)}$$

- Poimelli et al. (1989) – and others – suggested using the wall normal velocity

$$\tau_{i3,w}(x,y,t) = \langle\tau_w\rangle - C\langle\tau_w\rangle^{1/2}\tilde{w}(x+\Delta,y,t)$$

- Hultmark et al. (2013) suggested using velocity variance scaling to develop a local correction for the problem that the instantaneous log-law above won't preserve the mean shear stress value