LES of Turbulent Flows: Lecture 12

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Overview

1 A quick chat about Homework #2

2 Classes of LES models for τ_{ij}

3 Testing SGS Models

4 Eddy-Viscosity Models

A quick chat about Homework #2

The goal of the homework assignment:

- Calculate the 3D energy spectrum from isotropic data
- Perform 3D filtering on a 3D isotropic turbulence dataset
- Calculate the 3D energy spectrum of filtered data
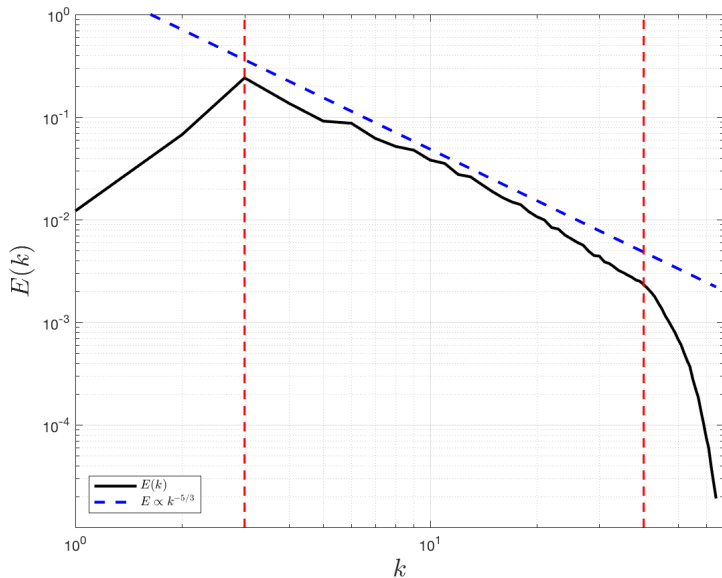- Compare unfiltered and filtered spectra

A quick chat about Homework #2

How to calculate the energy spectrum?
- Plot $E(|k|)$ vs. $|k| = \sqrt{[k_1^2 + k_2^2 + k_3^2]}$
- $E(|k|) = |\hat{u}_{\vec{k}}|^2 + |\hat{v}_{\vec{k}}|^2 + |\hat{w}_{\vec{k}}|^2$
- We estimate $E(|k|)$ by binning all the energy within a $d|k|$ shell in wavespace
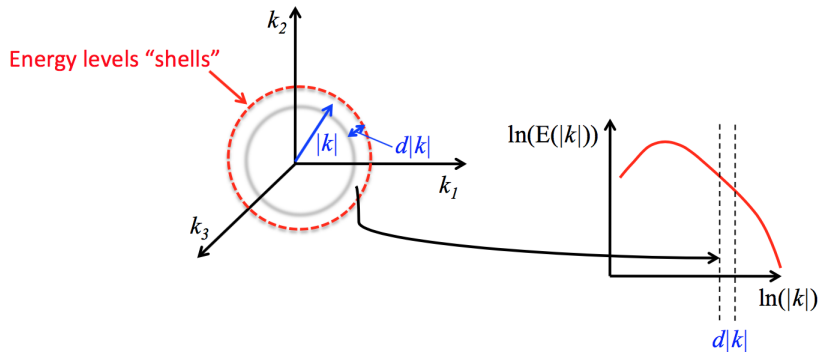
- Eddy-Viscosity Models
- Similarity Models
- Stochastic SGS Models
- Subgrid Velocity Reconstruction Models
- Dynamic Models

Eddy-Viscosity Models

- The largest and most commonly used class of SGS models.
- The general idea is that turbulent diffusion at SGSs (*i.e.* how SGSs remove energy) is analogous to molecular diffusion. It is very similar in form to K-theory for Reynolds stresses.

Eddy-Viscosity Models

- The deviatoric part of τ_{ij} is modeled as

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_T \widetilde{S}_{ij}$$

where ν_T is the SGS eddy-viscosity and

$$\widetilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i}\right)$$

is the strain rate tensor.

Within this class of models we have many different ways of determining ν_T

Eddy-Viscosity Models

- Smagorinsky model (Smagorinsky 1963)

- "Kolmolgorov" eddy-viscosity model (Wong and Lilly 1994)

- Two-point closure models (based on Kraichan's (1974) spectral eddy-viscosity model and developed by Lesiour (see Lesiour et al. 2005)

- One-equation models that use the filtered KE equation (see Lecture 7 for the KE equation). An early application of the model is found in Deardorff (1980). An overview of the Deardorff model and possible modifications are given in Gibbs and Fedorovich (2016)

Similarity Models

- These models were first introduced by Bardina et al. (1980)
- Based on the idea that the most active SGSs are those close to the cutoff wavenumber (or scale)
- A subclass under this type of model is the nonlinear model
- These models are often paired with an eddy-viscosity model

Stochastic SGS Models

- In these models a random (stochastic) component is incorporated into the SGS model
- The nonlinear term is usually combined with an eddy-viscosity model as is done in similarity models (Mason and Thomson 1992)

Subgrid Velocity Reconstruction Models

- These models seek to approximate the SGS stress through a direct reconstruction of the SGS velocity or scalar fields
- Two examples are:
    - fractal models (Scott and Meneveau 1999)
    - linear-eddy and one-dimensional turbulence models (Kerstein 1988)

Dynamic Models

- First developed by Germano (1991)
- Actually more of a procedure that can be applied to any base model
- One of the biggest and most influential ideas in LES SGS modeling

a posteriori testing

- Term can be credited to Piomelli et al. (1988)
- Run "full" simulations with a particular SGS model and compare the results (statistically) to DNS, experiments, or turbulence theory
- A "complete" test of the model (including dynamics and feedback)
- It has the disadvantage of including numerics and we can't gain insight into the physics of SGSs

a priori testing

- Use DNS or high resolution experimental data to test SGS models "offline"
- **Goal**: directly compare $\tau_{ij}^{\Delta}(\vec{x}, t)$ with $\tau_{ij}^{\Delta,\mathsf{M}}(\vec{x}, t)$
- How is this accomplished?

a priori testing

- Filter DNS (or experimental) data at Δ and compute the exact $\tau_{ij}^{\Delta}(\vec{x}, t)$ and other relevant parameters

- Use the filtered u, v, and w to compute $\tau_{ij}^{\Delta,\mathsf{M}}(\vec{x}, t)$ from the model (and stats) and compare with the above results. This can also be used to calculate unknown model coefficients

- Allows us to look specifically at how the model reproduces SGS properties and the physics associated with those properties

- Drawback – it does not include dynamic feedback and numerics!

Eddy-Viscosity Models

- See Guerts pg 225; Pope pg 587
- Eddy-viscosity models are of the form:

$$\text{momentum} \qquad \tau_{ij} = -2\nu_T \widetilde{S}_{ij}$$

$$\text{scalars} \qquad q_i = -D_T \frac{\partial \widetilde{\theta}}{\partial x_i}$$

where

$$D_T = \frac{\nu_T}{\mathsf{Pr}},$$

- $\widetilde{S}_{ij}$ is the filtered strain rate
- ν_T is eddy-viscosity
- D_T is eddy-diffusivity
- Pr is the SGS Prandtl number

Eddy-Viscosity Models

- This is the LES equivalent to 1st-order RANS closure (k-theory or gradient transport theory) and is an analogy to molecular viscosity (see Pope Ch. 10 for a review)

- Turbulent fluxes are assumed to be proportional to the local velocity or scalar gradients

- In LES, this is the assumption that stress is proportional to strain: $\tau_{ij} \sim \widetilde{S}_{ij}$

- The SGS eddy-viscosity ν_T must be parameterized

- Dimensionally

$$\nu_T = \left[\frac{L^2}{T}\right]$$

- In almost all SGS eddy-viscosity models:

$$\nu_T \sim U\ell$$

- Different models use different U and ℓ

- Recall the filtered N-S equations

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_j} + \underbrace{\frac{1}{\text{Re}}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2}}_{1} - \underbrace{\frac{\partial \tau_{ij}}{\partial x_j}}_{2} + F_i$$

- Let's focus on the viscous and subgrid stress terms

- Viscous term

$$\frac{1}{\mathsf{Re}}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} = \frac{1}{\mathsf{Re}}\frac{\partial}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j}\right) = \frac{1}{\mathsf{Re}}\frac{\partial}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

$$= \frac{2}{\mathsf{Re}}\frac{\partial \widetilde{S}_{ij}}{\partial x_j} - \frac{1}{\mathsf{Re}}\frac{\partial}{\partial x_j}\left(\frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

$$= \frac{2}{\mathsf{Re}}\frac{\partial \widetilde{S}_{ij}}{\partial x_j} - \frac{1}{\mathsf{Re}}\frac{\partial}{\partial x_i}\underbrace{\left(\frac{\partial \tilde{u}_j}{\partial x_j}\right)}_{}{}^{0}$$

$$= \frac{2}{\mathsf{Re}}\frac{\partial \widetilde{S}_{ij}}{\partial x_j}$$

- SGS stress term

$$\frac{\partial \tau_{ij}}{\partial x_j} = -\frac{\partial (2\nu_T \widetilde{S}_{ij})}{\partial x_j}$$

$$= -2\frac{\partial (\nu_T \widetilde{S}_{ij})}{\partial x_j}$$

- Combining the terms yields a new (approximate) viscous term

$$\frac{1}{\mathsf{Re}}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} = \boxed{2\frac{\partial}{\partial x_j}\left[\left(\nu_T + \frac{1}{\mathsf{Re}}\right)\widetilde{S}_{ij}\right]}$$

- If we were to write this in dimensional form:

$$\boxed{2\frac{\partial}{\partial x_j}\left[\left(\nu_T + \nu\right)\widetilde{S}_{ij}\right]}$$

  Thus, we can interpret the eddy-viscosity as adding to the molecular viscosity.

Eddy-Viscosity Models

- The eddy-viscosity extracts energy from the resolved scales in the simulation
- It mimics the average energy transfer in the turbulent cascade
- E-V model only transfer energy from resolved to subgrid – meaning the models only represent the statistically-averaged flow of energy and not the combined instantaneous forward scatter and backscatter

$$2\frac{\partial}{\partial x_j}\left[(\nu_T + \nu)\,\widetilde{S}_{ij}\right]$$

What does the model do?

- We can see it effectively lowers the Reynolds number of the flow

- It provides all of the energy dissipation for high Re flows (when $1/\text{Re} \Rightarrow 0$).

Most LES models use a nonlinear eddy viscosity. What happens if we use a constant?

- We effectively run the simulation at a different (lower) Re
- This has implications for DNS – if we try to use DNS at a lower Re to examine phenomena that happens at a higher Re, we can make the analogy between our DNS and an LES with a SGS model that doesn't properly reproduce the flow physics