Overview

1. Turbulence modeling (alternative strategies)
   Coherent Vortex Simulations (CVS)
   Filtered Density Functions (FDF)
Turbulence modeling (alternative strategies)

- Our discussion of turbulence modeling has centered around separating the flow into resolved and SFSs
- We have focused on using a low-pass filtering operation to accomplish this separation
- The goal of this procedure is to reduce the number of degrees of freedom in our numerical solution – i.e., make them more computationally affordable
- This is not the only way to accomplish complexity reduction in a turbulent flow
- We will briefly review a couple of different methods
• See Farge and Schneider (2001) on Canvas or class website

• The idea of CVS is that the turbulent flow field is decomposed into coherent (organized) and incoherent (random) components.

• The decomposition is accomplished by using either a continuous or orthonormal wavelet filter
• CVS, like LES, is classified as a semi-deterministic turbulence simulation method
• It is called semi-deterministic because some degrees of freedom are explicitly (deterministically) computed while the influence of others is modeled.
• DNS is considered fully deterministic since all scales of motion are resolved
• RANS is a fully statistical approach since only the steady solution of the mean flow field is solved deterministically, while the impact of all fluctuations are modeled
Figure: Methods to compute turbulent flows. Adapted from Farge and Schneider (2001).
Coherent Vortex Simulations (CVS)

- For CVS, a nonlinear wavelet filter is applied to the N-S equations
- Coherent vortices are extracted without the need to impose an a priori cut-off scale – unlike, e.g., LES
- The only a priori requirement is that the random (filtered-out) motions have a \( \sim \)Gaussian PDF
• In principle, the CVS approach deterministically solves the evolution of coherent vortices in a wavelet basis (we will review this in a minute)

• The wavelet basis adapts to regions of strong gradients

• Thus, CVS resolves the nonlinear interactions of coherent vortices

• Nonlinear vortex interactions lead to incoherent motions – these motions must be modeled
Coherent Vortex Simulations (CVS)

**Figure**: Total (left), coherent (center), and incoherent vorticity (right). From Farge and Schneider (2001).
Figure: Total (top), coherent (bottom left), and incoherent vorticity (bottom right). Adapted from Farge and Schneider (2001).
Coherent Vortex Simulations (CVS)

- In the original CVS formulation, the separation between coherent and random motions is assumed absolute, with the random part mimicking viscous dissipation
- Goldstein and Vasilyev (2004) introduced “stochastic coherent adaptive LES” as a variation on CVS
- GV04 use CVS wavelet decomposition, but do not assume that the wavelet filter completely eliminates all the coherent motions from the SFSs
- Thus, GV04 assumes that the SFS components themselves contain coherent and random components
• The CVS method uses wavelet decomposition
• We will cover a brief overview of wavelets. For a more detailed view see:
  • Daubechies (1992) – most recent printing is 2006
  • Mallat (2009) – 3rd edition
  • Farge (1992) – specific to turbulence research
A brief overview of wavelet decomposition

- With the normal Fourier transform (FT), we assume a periodic function.
- The FT only tells us what wavenumber (frequency) components exist in a signal.
- Space (time) and wavenumber (frequency) information cannot be seen at the same time.
- We need space-wavenumber (time-frequency) representation of a signal.
A brief overview of wavelet decomposition

- Why? Real-world signals are non-stationary, meaning it is useful to know if and where some feature happens
- A stationary signal has wavenumber (frequency) content unchanged in space (time), and all wavenumber (frequency) components exist everywhere (at all times)
- A non-stationary signal changes wavenumbers (frequency) in space (time)
A brief overview of wavelet decomposition

• One early solution is the Short Time (or space) Fourier Transform

• This technique only analyzes a small portion of signal at a time by using a space or time window – thus it is often called a Windowed Fourier Transform (WFT)

• The windowed segment is assumed stationary, and is applied uniformly
A brief overview of wavelet decomposition

- Recall, the FT is given by

\[ f_k = \frac{1}{2\pi} \int f(x) e^{ikx} \, dx \]

- The WFT is applied as

\[ f_{k,s} = \frac{1}{2\pi} \int f(s) g(s - x) e^{iks} \, ds \]

where \( s \) is the position over a localized region.
A brief overview of wavelet decomposition

- The WFT is our convolution with a filter function in Fourier space
A brief overview of wavelet decomposition

- There are drawbacks to this approach
  - The window is unchanged
  - The resolution dilemma – a narrow window has poor wavenumber (frequency) resolution, and a wide window has poor spatial (time) resolution
  - Heisenberg Uncertainty Principle – we cannot know what wavenumber (frequency) exists at what spatial (time) intervals
A brief overview of wavelet decomposition

- Another approach is called wavelet decomposition
- A wavelet is just some small wave
- The general idea is to decompose a signal into series of wavelets
- This approach helps overcome the resolution dilemma of WFTs
A brief overview of wavelet decomposition

- Wavelets offer an optimal space/frequency decomposition.

\[ W_f(a, b) = |a|^{-\frac{1}{2}} \int f(x) \Psi \left( \frac{x - b}{a} \right) dx \]

where \( \Psi \) is the basis function ("Mother Wavelet"), \( b \) translates the basis function, and \( a \) scales (dilates) the basis function

- A mother wavelet is a prototype for generating the other window functions
A brief overview of wavelet decomposition

Figure: from wavelet.org
Common properties of wavelets

- A wavelet transform is a set of building blocks to construct or represent a signal (function)
- A wavelet is a 2D expansion set (usually a basis) for a 1D signal
- A wavelet expansion gives a space-wavenumber (time-frequency) localization of a signal
- The calculation of coefficients can be done efficiently
Common properties of wavelets

- Wavelet systems are generated from a single scaling function (i.e. wavelet) by simple scaling and translation.
- Most useful wavelet systems satisfy the multiresolution condition – if the basic expansion signals (the wavelets) are made half as wide and translated in steps half as wide, they will represent a larger class of signals exactly or give a better approximation of any signal.
- The lower resolution coefficients can be calculated from the higher resolution coefficients by a tree-structured algorithm.
Multiresolution properties of wavelets

- Analyze the signal at different frequencies with different resolutions
- Good time resolution and poor frequency resolution at high frequencies
- Good frequency resolution and poor time resolution at low frequencies
- More suitable for short duration of higher frequency; and longer duration of lower frequency components
A brief overview of wavelet decomposition

- One example of a wavelet is the Haar wavelet (Haar, 1910)

\[
\Psi = \begin{cases} 
1 & \text{if } 0 \leq x < 0.5 \\
-1 & \text{if } 0.5 \leq x < 1
\end{cases}
\]
A brief overview of wavelet decomposition

- How does wavelet decomposition break down a signal in space and time?
A brief overview of wavelet decomposition

- The signal is reconstructed by combining $s_i$ and $d_i$ at the desired level ($s$, $ss$, $sss$, etc).
  - 1st level – $s_i$ and $d_i$
  - 2nd level – $ss_i$, $dd_i$, $d_i$
  - ...

![Wavelet “tree” decomposition diagram](image)

For the Haar wavelet

- $s_1 = \frac{(x_1 + x_2)}{2^{\frac{1}{2}}}$
- $d_1 = \frac{(x_2 - x_1)}{2^{\frac{1}{2}}}$
Filtered Density Functions (FDF)

- See Colucci et al. (1998) on Canvas or class website
- In this method, the evolution of the filtered probability density functions is solved for (i.e., we solve for the evolution of the SFS general moments)
- Similar to general PDF transport methods 1st introduced by Lundgren (1969) and outlined in detail in Pope chapter 12.
Filtered Density Functions (FDF)

- Many applications use FDF for scalars in turbulent reacting flows, while traditional equations (low-pass filtered N-S) are solved for momentum (see Fox 2012 for a more detailed discussion)
- This type of method is open employed for LES with Lagrangian particle models and for chemically reactive flows
- In Lagrangian particle models it leads to a form of the Langevin equations for SFS particle evolution
- In chemically reactive flows it has the advantage that the reactions occur in closed form
- We will return to these type of methods later in the class when we discuss combining LES with particle models