# LES of Turbulent Flows: Lecture 10

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# Overview

1. Turbulence modeling (alternative strategies)
    - Coherent Vortex Simulations (CVS)
    - Filtered Density Functions (FDF)

# Turbulence modeling (alternative strategies)

- Our discussion of turbulence modeling has centered around separating the flow into resolved and SFSs
- We have focused on using a low-pass filtering operation to accomplish this separation
- The goal of this procedure is to reduce the number of degrees of freedom in our numerical solution – *i.e.*, make them more computationally affordable
- This is not the only way to accomplish complexity reduction in a turbulent flow
- We will briefly review a couple of different methods

# Coherent Vortex Simulations (CVS)

- See Farge and Schneider (2001) on Canvas or class website
- The idea of CVS is that the turbulent flow field is decomposed into coherent (organized) and incoherent (random) components.
- The decomposition is accomplished by using either a continuous or orthonormal wavelet filter

# Coherent Vortex Simulations (CVS)

- CVS, like LES, is classified as a semi-deterministic turbulence simulation method
- It is called semi-deterministic because some degrees of freedom are explicitly (deterministically) computed while the influence of others is modeled.
- DNS is considered fully deterministic since all scales of motion are resolved
- RANS is a fully statistical approach since only the steady solution of the mean flow field is solved deterministically, while the impact of all fluctuations are modeled

# Coherent Vortex Simulations (CVS)

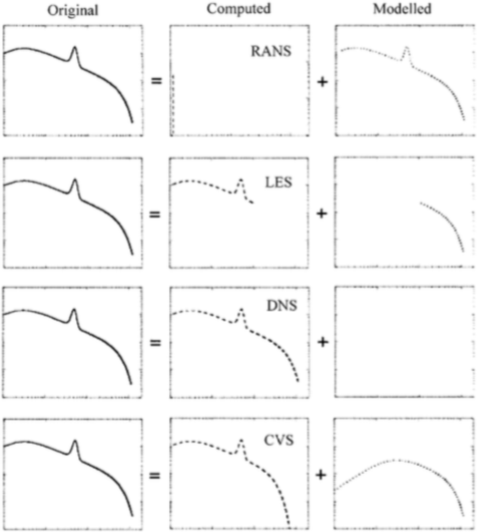



Figure: Methods to compute turbulent flows. Adapted from Farge and Schneider (2001).

# Coherent Vortex Simulations (CVS)

- For CVS, a nonlinear wavelet filter is applied to the N-S equations
- Coherent vortices are extracted without the need to impose an *a priori* cut-off scale – unlike, *e.g.*, LES
- The only *a priori* requirement is that the random (filtered-out) motions have a $\sim$Gaussian PDF

# Coherent Vortex Simulations (CVS)

- In principle, the CVS approach deterministically solves the evolution of coherent vortices in a wavelet basis (we will review this in a minute)
- The wavelet basis adapts to regions of strong gradients
- Thus, CVS resolves the nonlinear interactions of coherent vortices
- Nonlinear vortex interactions lead to incoherent motions – these motions must be modeled

# Coherent Vortex Simulations (CVS)

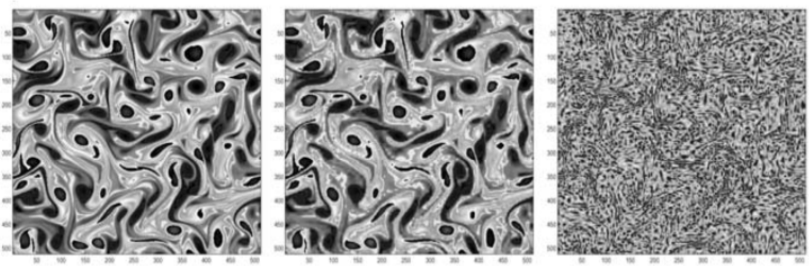


Figure: Total (left), coherent (center), and incoherent vorticity (right). From Farge and Schneider (2001).

# Coherent Vortex Simulations (CVS)

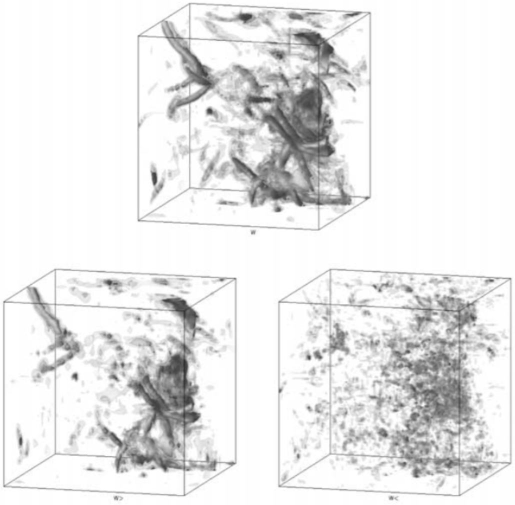


Figure: Total (top), coherent (bottom left), and incoherent vorticity (bottom right). Adapted from Farge and Schneider (2001).

# Coherent Vortex Simulations (CVS)

- In the original CVS formulation, the separation between coherent and random motions is assumed absolute, with the random part mimicking viscous dissipation
- Goldstein and Vasilyev (2004) introduced "stochastic coherent adaptive LES" as a variation on CVS
- GV04 use CVS wavelet decomposition, but do not assume that the wavelet filter completely eliminates all the coherent motions from the SFSs
- Thus, GV04 assumes that the SFS components themselves contain coherent and random components

# A brief overview of wavelet decomposition

- The CVS method uses wavelet decomposition
- We will cover a brief overview of wavelets. For a more detailed view see:
  - Daubechies (1992) – most recent printing is 2006
  - Mallat (2009) – 3rd edition
  - Farge (1992) – specific to turbulence research

# A brief overview of wavelet decomposition

- With the normal Fourier transform (FT), we assume a periodic function
- The FT only tells us what wavenumber (frequency) components exist in a signal
- Space (time) and wavenumber (frequency) information cannot be seen at the same time
- We need space-wavenumber (time-frequency) representation of a signal

# A brief overview of wavelet decomposition

- Why? Real-world signals are non-stationary, meaning it is useful to know if and where some feature happens
- A stationary signal has wavenumber (frequency) content unchanged in space (time), and all wavenumber (frequency) components exist everywhere (at all times)
- A non-stationary signal changes wavenumbers (frequency) in space (time)

# A brief overview of wavelet decomposition

- One early solution is the Short Time (or space) Fourier Transform
- This technique only analyzes a small portion of signal at a time by using a space or time window – thus it is often called a Windowed Fourier Transform (WFT)
- The windowed segment is assumed stationary, and is applied uniformly

# A brief overview of wavelet decomposition

- Recall, the FT is given by

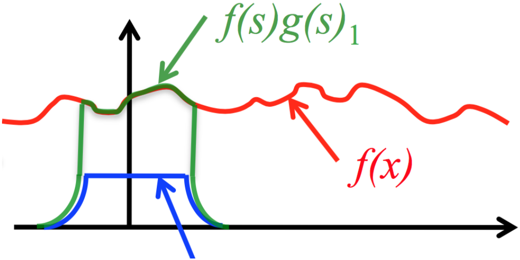
$$f_k = \frac{1}{2\pi}\int f(x)e^{ikx}dx$$

- The WFT is applied as

$$f_{k,s} = \frac{1}{2\pi}\int f(s)g(s-x)e^{iks}ds$$

where $s$ is the position over a localized region

# A brief overview of wavelet decomposition




- The WFT is our convolution with a filter function in Fourier space

# A brief overview of wavelet decomposition

- There are drawbacks to this approach
  - The window is unchanged
  - The resolution dilemma – a narrow window has poor wavenumber (frequency) resolution, and a wide window has poor spatial (time) resolution
  - Heisenberg Uncertainty Principle – we cannot know what wavenumber (frequency) exists at what spatial (time) intervals

# A brief overview of wavelet decomposition

- Another approach is called wavelet decomposition
- A wavelet is just some small wave
- The general idea is to decompose a signal into series of wavelets
- This approach helps overcome the resolution dilemma of WFTs

# A brief overview of wavelet decomposition

- Wavelets offer an optimal space/frequency decomposition.

$$W_f(a,b) = |a|^{-\frac{1}{2}} \int f(x)\Psi\left(\frac{x-b}{a}\right)dx$$

  where $\Psi$ is the basis function ("Mother Wavelet"), $b$ translates the basis function, and $a$ scales (dilates) the basis function
- A mother wavelet is a prototype for generating the other window functions

# A brief overview of wavelet decomposition

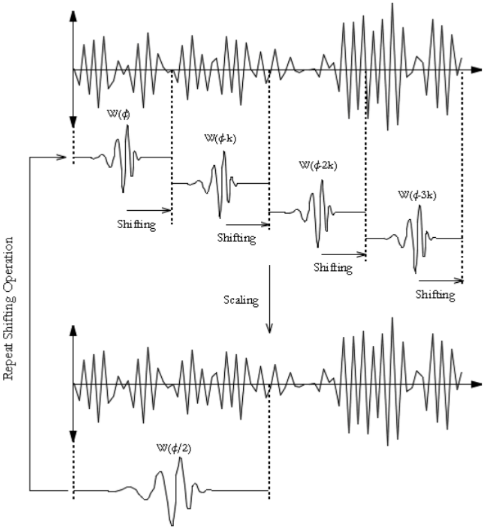



Figure: from wavelet.org

# A brief overview of wavelet decomposition

Common properties of wavelets

- A wavelet transform is a set of building blocks to construct or represent a signal (function)
- A wavelet is a 2D expansion set (usually a basis) for a 1D signal
- A wavelet expansion gives a space-wavenumber (time-frequency) localization of a signal
- The calculation of coefficients can be done efficiently

# A brief overview of wavelet decomposition

Common properties of wavelets

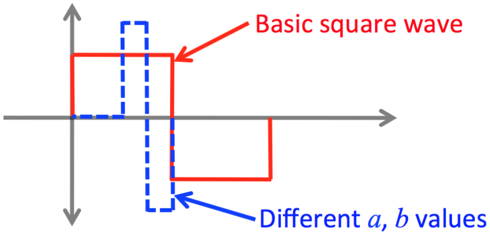
- Wavelet systems are generated from a single scaling function (*i.e.* wavelet) by simple scaling and translation
- Most useful wavelet systems satisfy the multiresolution condition – if the basic expansion signals (the wavelets) are made half as wide and translated in steps half as wide, they will represent a larger class of signals exactly or give a better approximation of any signal
- The lower resolution coefficients can be calculated from the higher resolution coefficients by a tree-structured algorithm

# A brief overview of wavelet decomposition

Multiresolution properties of wavelets

- Analyze the signal at different frequencies with different resolutions
- Good time resolution and poor frequency resolution at high frequencies
- Good frequency resolution and poor time resolution at low frequencies
- More suitable for short duration of higher frequency; and longer duration of lower frequency components

# A brief overview of wavelet decomposition

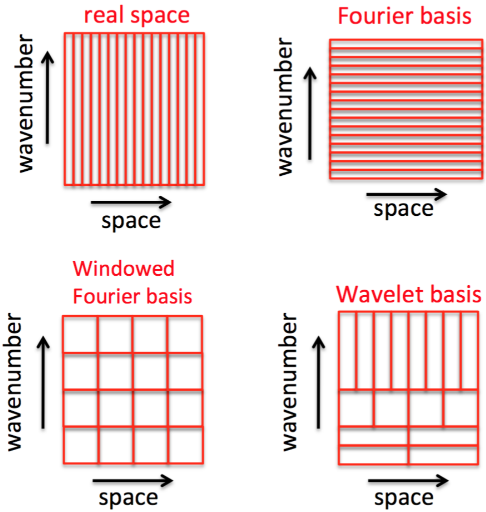



- One example of a wavelet is the Haar wavelet (Haar, 1910)

$$\Psi = \begin{cases} 1 & \text{if } 0 \leq x < 0.5 \\ -1 & \text{if } 0.5 \leq x < 1 \end{cases}$$

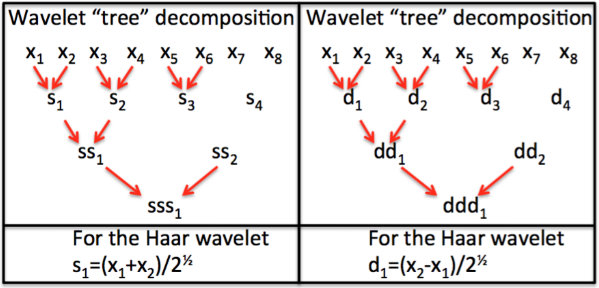
# A brief overview of wavelet decomposition

- How does wavelet decomposition break down a signal in space and time?



# A brief overview of wavelet decomposition

- The signal is reconstructed by combining $s_i$ and $d_i$ at the desired level ($s$, $ss$, $sss$, etc).
  - 1st level – $s_i$ and $d_i$
  - 2nd level – $ss_i$, $dd_i$, $d_i$
  - ...



# Filtered Density Functions (FDF)

- See Colucci et al. (1998) on Canvas or class website
- In this method, the evolution of the filtered probability density functions is solved for (*i.e.*, we solve for the evolution of the SFS general moments)
- Similar to general PDF transport methods 1st introduced by Lundgren (1969) and outlined in detail in Pope chapter 12.

# Filtered Density Functions (FDF)

- Many applications use FDF for scalars in turbulent reacting flows, while traditional equations (low-pass filtered N-S) are solved for momentum (see Fox 2012 for a more detailed discussion)
- This type of method is open employed for LES with Lagrangian particle models and for chemically reactive flows
- In Lagrangian particle models it leads to a form of the Langevin equations for SFS particle evolution
- In chemically reactive flows it has the advantage that the reactions occur in closed form
- We will return to these type of methods later in the class when we discuss combining LES with particle models