Large-Eddy Simulation of Turbulent Flows

ME EN 7960-003 | Homework #1 | Due: September 20th

1.) Pope Exercise 3.1

With Q and R being random variables, and a and b, being constants use the following equation

$$\langle Q(U)\rangle \equiv \int_{-\infty}^{\infty} Q(V) f(V) dV$$

to verify that:

(a) $\langle a\rangle = a$ and $\langle aQ\rangle = a\langle Q\rangle$

(b) $\langle Q + R\rangle = \langle Q\rangle + \langle R\rangle$ and $\langle\langle Q\rangle\rangle = \langle Q\rangle$

(c) $\langle\langle Q\rangle\langle R\rangle\rangle = \langle Q\rangle\langle R\rangle$ and $\langle\langle Q\rangle R\rangle = \langle Q\rangle\langle R\rangle$

(d) $\langle q'\rangle = 0$ and $\langle q'\langle R\rangle\rangle = 0$, where $q' = Q - \langle Q\rangle$

Note: it is acceptable to use a rule (make sure to refer to it) after you have proven it.

2.) Pope Exercise 3.2

Let Q be defined by $Q = a + bU$, where U is a stationary random variable and a and b are constants. Show that:

(a) $\langle Q\rangle = a + b\langle U\rangle$

(b) $\text{var}(Q) = b^2 \text{ var}(U)$

(c) $\text{sdev}(Q) = b \text{ sdev}(U)$

(d) $\text{var}(U) = \langle U^2\rangle - \langle U\rangle^2$

3.) Probability and statistics

Using the data set *v1obsdata.txt*, located at http://gibbs.science/les/homework/v1obsdata.txt or on Canvas, calculate and plot (if required) the following:

(a) Calculate the probability density function (PDF) of V_1 and plot it using an appropriate bin size (your choice). Describe why you chose your bin size.

(b) Calculate the first *four* moments (mean, variance, skewness, and kurtosis) of the data and describe how each one is linked to the PDF you plotted in part (a). Note that you can calculate the moments from discrete estimates.

(c) Calculate and plot the autocorrelation function of the data. Comment on how well you can estimate the integral time scale from your plot.

4.) Power spectra

Using the data set *v1simdata.txt*, located at http://gibbs.science/les/homework/v1simdata.txt or on Canvas, calculate and plot the 1D power spectral density as a function of wavenumber. Note that the data is a 2D horizontal slice from a 3D turbulence simulation. Calculate the power spectra in the streamwise (x) direction and then average the spectra over the spanwise direction so that your final estimation is:

$$\langle E_{11}(k_1)\rangle = \frac{1}{N_y}\sum_{j=1}^{N_y} \hat{f}_{k_1}\hat{f}^*_{k_1}$$

The data is saved in the file as ASCII text data with three columns. Column 1 is the x streamwise position in meters, column 2 is the y spanwise position in meters, and column 3 is the V_1 streamwise velocity in m/s. The 2D field has 256 points in each direction with $N_x = N_y = 256$. Make *two* plots of the spectra (E11 vs k1): one with linear/linear scaling and a second with log/log scaling. Comment on any observations you can make about the shape of the spectra.