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Evaluation of dynamic subgrid-scale models in large-eddy simulations of neutral turbulent flow over a two-dimensional sinusoidal hill

Feng Wan, Fernando Porté-Agel*, Rob Stoll

Saint Anthony Falls Laboratory, Department of Civil Engineering, University of Minnesota Twin Cities, Minneapolis, MN, USA

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Abstract

Large-eddy simulation (LES) is used to simulate neutral turbulent boundary-layer flow over a rough two-dimensional sinusoidal hill. Three different subgrid-scale (SGS) models are tested: (a) the standard Smagorinsky model with a wall-matching function, (b) the Lagrangian dynamic model, and (c) the recently developed scale-dependent Lagrangian dynamic model [Stoll, R., Porté-Agel, F., 2006. Dynamic subgrid-scale models for momentum and scalar fluxes in large-eddy simulation of neutrally stratified atmospheric boundary layers over heterogeneous terrain. Water Resources Research 42, W01409. doi:10.1029/2005WR003989]. The simulation results obtained with the different models are compared with turbulence statistics obtained from experiments conducted in the meteorological wind tunnel of the AES (Atmospheric Environment Service, Canada) [Gong, W., Taylor, P.A., Dörnbrack, A., 1996. Turbulent boundary-layer flow over fixed aerodynamically rough two-dimensional sinusoidal waves. Journal of Fluid Mechanics 312, 1–37]. We find that the scale-dependent dynamic model is able to account, without any tuning, for the local changes in the eddy-viscosity model coefficient. It can also capture the scale dependence of the coefficient associated with regions of the flow with strong mean shear and flow anisotropy. As a result, the scale-dependent dynamic model yields results that are more realistic than the ones obtained with the scale-invariant Lagrangian dynamic model.

Keywords: Large-eddy simulation; Subgrid-scale modeling; Two-dimensional sinusoidal hill

1. Introduction

Large-eddy simulation (LES) can provide valuable high resolution spatial and temporal information necessary to understand the effects of topography on turbulent transport in the atmospheric boundary layer (ABL) (e.g., Krettenauer and Schumann, 1992; Walko et al., 1992; Dörnbrack and Schumann, 1993; Maass and Schumann, 1994; Gong et al., 1996; Henn and Sykes, 1999; Brown et al., 2001; Iizuka and Kondo, 2004). LES explicitly resolves all scales of turbulent transport larger than the grid scale Δ (on the order of tens of meters in the ABL), while the smallest (less energetic) scales are parameterized using a subgrid-scale (SGS) model. Despite the potential of LES, however, the strong spatial heterogeneity and

*Corresponding author. Tel.: +1 612 625 8055.
E-mail address: fporte@umn.edu (F. Porté-Agel).

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flow anisotropy associated with topography hinder the performance of commonly used subgrid-scale models (e.g., Iizuka and Kondo, 2004, 2006).

Eddy-viscosity models are the most commonly used SGS models in LES of ABL flows. One of the main challenges in the implementation of these models is the specification of the model coefficients. Although the coefficients are well established for the case of isotropic turbulence (e.g. Smagorinsky, 1963; Lilly, 1967; Germano et al., 1991), there is evidence from both simulations and experimental a priori studies that these coefficients should decrease in regions of the flow with large flow anisotropy at the smallest resolved and subgrid scales associated with large local mean shear (Deardorff, 1971, 1980; Hunt et al., 1988; Schumann, 1991; Horiuti, 1993; Canuto and Cheng, 1997; Porté-Agel et al., 2001; Kleissl et al., 2003; Porté-Agel, 2004; Stoll and Porté-Agel, 2006).

One systematic approach to account for the spatial and temporal variability of the SGS model coefficient is the use of dynamic procedures (Germano et al., 1991; Ghosal et al., 1995; Meneveau et al., 1996), which consist of optimizing the value of the model coefficient at every position and time step by using information contained in the resolved scales and assuming scale invariance of the coefficient between the filter/grid scale and a slightly larger, test-filter scale. In order to guarantee numerical stability, these procedures require some kind of averaging. If the flow has directions of homogeneity, the averaging can be done over those directions (e.g., over horizontal planes in the case of flow over a flat homogeneous surface). For cases of flow over complex terrain, Lagrangian averaging (over flow pathlines) has been used in dynamic models (Meneveau et al., 1996). The dynamic model has been found to yield unrealistic turbulence statistics (e.g. mean velocity profiles and turbulence spectra) in simulations of ABL flows over homogeneous (Porté-Agel et al., 2000) and heterogeneous (Bou-Zeid et al., 2005; Stoll and Porté-Agel, 2006) flat surfaces, as well as flows over topography (Iizuka and Kondo, 2004). Iizuka and Kondo (2004) tested the dynamic model and the Lagrangian dynamic model in simulations of a turbulent boundary layer over a two-dimensional hill. In comparisons with experimental wind-tunnel measurements of Ishihara and Hibi (1998), the simulation results of Iizuka and Kondo (2004) showed that the standard dynamic and Lagrangian dynamic models overestimated the time-averaged velocity near the surface over the hill crest. Porté-Agel et al. (2000) showed that in simulations of neutral boundary layers over homogeneous surfaces, the dynamically computed coefficients are scale dependent, which is inconsistent with the assumption of scale invariance on which the dynamic procedure relies. Motivated by their results, Porté-Agel et al. (2000) introduced the so-called scale-dependent dynamic model by relaxing the assumption of scale invariance of the model coefficient. The scale-dependent dynamic model was shown to overcome the limitations of the scale-invariant dynamic model in simulations of neutral ABL flows over flat surfaces. More recently, a scale-dependent Lagrangian dynamic model version has been used in simulations of flow over heterogeneous surfaces (Bou-Zeid et al., 2005; Stoll and Porté-Agel, 2006). The performance of this model over topography has not been tested to date.

In this study, large-eddy simulation is used to simulate an experimentally well characterized turbulent boundary layer flow over a two-dimensional sinusoidal hill. Three different SGS models are tested: (a) the standard Smagorinsky model with a wall-matching function, (b) the Lagrangian dynamic model, and (c) the recently developed scale-dependent Lagrangian dynamic model (Stoll and Porté-Agel, 2006). The simulation results obtained with the different models are compared with turbulence statistics obtained from experiments conducted in the meteorological wind tunnel of the AES (Atmospheric Environment Service, Canada) (Gong et al., 1996). Next, a brief description of the three models is given.

1.1. The Smagorinsky model

The eddy-viscosity model is commonly used in LES to parameterize the SGS stresses τ_{ij} as

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_T\tilde{S}_{ij}, \tag{1}$$

where the tilde denotes spatial filtering using a three-dimensional filter of size Δ, $\tilde{S}_{ij} = 1/2\left((\partial\tilde{u}_i/\partial x_j) + (\partial\tilde{u}_j/\partial x_i)\right)$ is the resolved (filtered) strain rate tensor, and ν_T is the eddy viscosity, which is defined as (Smagorinsky, 1963)

$$\nu_T = [C_S\Delta]^2|\tilde{S}|, \tag{2}$$

where $|\tilde{S}| = (2\tilde{S}_{ij}\tilde{S}_{ij})^{1/2}$ is the magnitude of the resolved strain-rate tensor, and C_S is a non-dimensional parameter called the Smagorinsky coefficient. The value of the model parameter C_S

is well established for isotropic, homogeneous turbulence with cutoff in the inertial subrange and Δ equal to the grid size ($C_S \sim 0.17$, Lilly, 1967). However, anisotropy of the flow due to strong mean shear near the surface makes the optimum value of C_S depart from its isotropic counterpart. A decrease in the Smagorinsky coefficient is associated with an increase in anisotropy in both the resolved and SGS velocities (Deardorff, 1971, 1980; Hunt et al., 1988; Schumann, 1991; Horiuti, 1993; Canuto and Cheng, 1997; Porté-Agel et al., 2001; Kleissl et al., 2003; Porté-Agel, 2004; Stoll and Porté-Agel, 2006). In order to account for these effects, application of eddy-diffusion models in LES of the ABL has often involved the use of various types of *ad hoc* wall damping corrections. For example, Mason and Thomson (1992) proposed to use the equation

$$\frac{1}{\lambda^n} = \frac{1}{\lambda_0^n} + \frac{1}{[\kappa(z+z_0)]^n}, \tag{3}$$

where $\kappa (\approx 0.4)$ is the von Karman constant, $\lambda = C_S\Delta$ is the length scale in the model, $\lambda_0 = C_0\Delta$ is the length scale far from the wall, z_0 is the roughness length, and C_0 and n are adjustable parameters. They apply this formulation with different values of C_0 (from about 0.1 to 0.3) and n (1, 2, and 3).

1.2. The Lagrangian dynamic model

The dynamic procedure (Germano et al., 1991) provides a systematic way to calculate the value of the model coefficient (C_S^2) at every time and position in the flow based on the dynamics of the smallest resolved scales. The model is based on the Germano identity

$$L_{ij} = T_{ij} - \bar{\tau}_{ij} = \overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j, \tag{4}$$

where L_{ij} is the so-called Leonard stress tensor that can be calculated based on the resolved scales, and T_{ij} is the SGS stress at a test filter scale $\bar{\Delta}$ (typically $\bar{\Delta} = 2\Delta$). The overbar denotes a spatial filtering operation at scale $\bar{\Delta}$.

Applying the Smagorinsky model at the test filter scale yields the following equation:

$$T_{ij} - \tfrac{1}{3}\delta_{ij}T_{kk} = -2(C_S(\bar{\Delta})\bar{\Delta})^2|\bar{\tilde{S}}|\bar{\tilde{S}}_{ij}. \tag{5}$$

Substituting Eqs. (1) and (5) into (4) and assuming scale invariance of the model coefficient, i.e.,

$$C_S^2(2\Delta) = C_S^2(\Delta), \tag{6}$$

results in the equations describing the error associated with the use of the Smagorinsky model in the Germano identity

$$e_{ij} = L_{ij} - \tfrac{1}{3}\delta_{ij}L_{kk} - C_S^2(\Delta)M_{ij}, \tag{7}$$

where

$$M_{ij} = 2\Delta^2\left(\overline{|\tilde{S}|\tilde{S}_{ij}} - 4|\bar{\tilde{S}}|\bar{\tilde{S}}_{ij}\right). \tag{8}$$

Optimizing the value of C_S^2 through least squares minimization of the error given by Eq. (7) (Lilly, 1992; Ghosal et al., 1995) leads to

$$C_S^2 = \frac{\langle L_{ij}M_{ij}\rangle}{\langle M_{ij}M_{ij}\rangle}. \tag{9}$$

In order to implement the dynamic model, some sort of averaging (denoted with brackets $\langle\,\rangle$ in Eq. (9)) needs to be used to guarantee numerical stability of the procedure. Typically averaging is done over directions of flow homogeneity (e.g., horizontal planes over flat homogeneous terrain), or over flow pathlines using the Lagrangian averaging procedure developed by Meneveau et al. (1996). Lagrangian dynamic models are therefore suitable for simulations of the ABL over complex terrain, where there is no direction of homogeneity in the flow. More detailed descriptions of the Lagrangian dynamic procedure can be found in Stoll and Porté-Agel (2006).

The dynamic model avoids the need for a priori specification or tuning of the coefficient because it is evaluated directly from the resolved scales in the LES. However, recent studies have shown that the dynamic models have problems reproducing the correct flow statistics over both flat surfaces (Porté-Agel et al., 2000) as well as complex terrain (Iizuka and Kondo, 2004).

1.3. The scale-dependent Lagrangian dynamic model

Recently, Porté-Agel et al. (2000) proposed a scale-dependent dynamic model, a modification of the dynamic procedure that allows the model coefficient to change with scale (i.e. not assuming that $C_S^2(\Delta) = C_S^2(2\Delta)$). We can still write down the Germano identity for the Smagorinsky model. However, now M_{ij} also depends on the ratio of the model coefficient at the test filter scale and the filter scale (Meneveau and Lund, 1997) and can be expressed as

$$M_{ij} = 2\Delta^2\left(\overline{|\tilde{S}|\tilde{S}_{ij}} - 4\beta|\bar{\tilde{S}}|\bar{\tilde{S}}_{ij}\right). \tag{10}$$

Note that Eq. (10) includes a new variable, the scale-dependence parameter $\beta = C_S^2(2\Delta)/C_S^2(\Delta)$. To obtain a dynamic value for β, we use a second test filter at another scale larger than Δ, e.g. $\hat{\Delta} = 4\Delta$, and denote variables filtered at scale 4Δ by a caret. By using the second test filter, the error associated with the use of the Smagorinsky model in the Germano identity between Δ and $\hat{\Delta}$ now becomes

$$e'_{ij} = L'_{ij} - \tfrac{1}{3}\delta_{ij} L'_{kk} - C_S^2(\Delta) M'_{ij}, \tag{11}$$

where

$$L'_{ij} = \widehat{\tilde{u}_i \tilde{u}_j} - \hat{\tilde{u}}_i \hat{\tilde{u}}_j \tag{12}$$

$$M'_{ij} = 2\Delta^2 \left(\widehat{|\tilde{S}|\tilde{S}_{ij}} - 4^2\beta^2 |\hat{\tilde{S}}| \hat{\tilde{S}}_{ij} \right). \tag{13}$$

At this point, some assumption has to be made about the functional form of the scale dependence of the coefficient. Porté-Agel et al. (2000) assumed that C_S^2 can be expressed as a power-law function of Δ, which implies

$$\beta = C_S^2(2\Delta)/C_S^2(\Delta) = C_S^2(4\Delta)/C_S^2(2\Delta) \tag{14}$$

and therefore

$$\beta^2 = \frac{C_S^2(4\Delta)}{C_S^2(\Delta)}. \tag{15}$$

It is important to note that the power-law scaling assumption for C_S^2 is much weaker than the previous assumption of scale invariance (i.e., $\beta = 1$) on which the original (scale-invariant) dynamic model relies.

The same method used with the first test filter is employed here to minimize the error in Eq. (11) locally backward along the fluid path line, resulting in the following equation for $C_S^2(\mathbf{x}, t)$:

$$C_S^2(\mathbf{x}, t) = \frac{\left\langle L'_{ij} M'_{ij} \right\rangle}{\left\langle M'_{ij} M'_{ij} \right\rangle}, \tag{16}$$

where, in the case of the scale-dependent Lagrangian dynamic model, the brackets $\langle\,\rangle$ denote averaging along fluid pathlines. Setting Eq. (16) equal to Eq. (9) results in a single equation from which the unknown scale dependence parameter $\beta(\mathbf{x},\ t)$ may be obtained dynamically. For more details on the scale-dependent Lagrangian dynamic procedure, see Stoll and Porté-Agel (2006).

By using information on the dynamics of the flow corresponding to an additional test-filter scale (e.g. 4Δ), the scale-dependent model has the ability to detect and account for scale dependence in a dynamic manner (based on the information of the resolved field and, thus, not requiring any tuning of parameters). In particular, the scale-dependent dynamic model is used to dynamically calculate not only $C_S^2(\Delta)$, but also the value of the scale-dependence coefficient $\beta = C_S^2(2\Delta)/C_S^2(\Delta)$. Scale-dependent Lagrangian dynamic models have successfully been implemented in simulations of ABLs over flat heterogeneous terrain (Bou-Zeid et al., 2005; Stoll and Porté-Agel, 2006). In this paper, we study the performance of the scale-dependent Lagrangian dynamic model in simulations of a boundary layer over rough two-dimensional sinusoidal hills.

2. Numerical experiments

The large-eddy simulation code is a modified version of the code described by Albertson and Parlange (1999), Porté-Agel et al. (2000), and Stoll and Porté-Agel (2006). The code uses a mixed pseudospectral finite-difference method, i.e., spatial derivatives are computed using pseudospectral methods in the horizontal directions and finite differences in the vertical direction. Consequently, the boundary conditions in the horizontal directions are periodic. A second-order Adams-Bashforth scheme is used for time advancement. The upper boundary condition is a fixed stress-free lid. The lower boundary condition consists of using similarity theory (the logarithmic law) to calculate the instantaneous (filtered) surface shear stress as a function of the velocity field at the lowest computational level. In particular, the two components of the surface shear stress vector are calculated following:

$$\tau_{xz} = -C_d \tilde{V}(\tilde{u} \cos\theta_x + \tilde{w} \sin\theta_x), \tag{17}$$

$$\tau_{yz} = -C_d \tilde{V}(\tilde{v} \cos\theta_y + \tilde{w} \sin\theta_y), \tag{18}$$

where C_d is the drag coefficient obtained from the logarithmic law. $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ are the filtered streamwise, spanwise and vertical velocities, and $\tilde{V}$ is the magnitude of the tangential velocity, all calculated at the lowest computational grid level. θ_x and θ_y are the local angles of inclination of the topography in the x and y direction, respectively ($\theta_y = 0$, in our case).

The simulated physical domain corresponds to the space above two sinusoidal waves with non-dimensional elevation:

$$z_s/L_z = a \cos(2x/L_z), \tag{19}$$

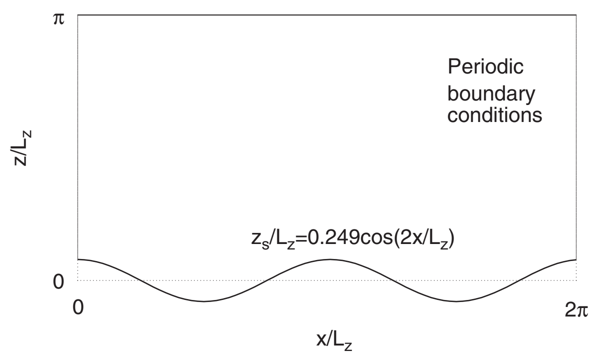

Fig. 1. Schematic of computational domain over the sinusoidal hill.

where $a = 0.249$ is the normalized wave amplitude, x/L_z is the normalized streamwise position, and L_z is the length scale used for normalization (see Fig. 1). The flow direction is perpendicular to the wave crests. The coordinate transformation developed by Clark (1977) has been used to transform the sinusoidal wave bounded physical domain into a rectangular computational domain. The transformation is a terrain following transformation and takes the following form:

$$\bar{z} = H(z - z_s)/(H - z_s), \tag{20}$$

where $\bar{z}$ is the vertical position in the transformed system. z_s and H denote the actual elevation (in the original system) of the terrain and the top of the domain, respectively. In order to match the wind-tunnel experimental conditions of Gong et al. (1996), the computational domain, after normalization with the length scale $L_z = 194\,\text{mm}$, is of size $(2\pi, 2\pi, \pi)$. The non-dimensional surface roughness is set to $z_0/L_z = 2.06 \times 10^{-3}$. The computational domain is divided into $80 \times 80 \times 80$ uniformly spaced grid points. The grid is staggered in the vertical direction, with the vertical velocity stored halfway between the other variables. Wind velocities are normalized using the free stream wind tunnel velocity, $U_0 = 10\,\text{m/s}$.

A horizontal pressure gradient is exerted on the flow in the streamwise direction. The magnitude of this pressure gradient is set to balance the drag forces (surface stress and form drag) measured during the experiment (Gong et al., 1996). The value of the non-dimensional pressure gradient is 0.654. The simulations are run for a period of time long enough to guarantee quasi-steady flow conditions and statistical convergence of the results presented in the next section.

3. Results

Fig. 2a–d show the simulated non-dimensional streamwise velocity profiles obtained above four different streamwise positions in a wave: the wave crest (Fig. 2a), 1/4 wavelength downwind of the crest (Fig. 2b), the wave trough (Fig. 2c), and 1/4 wavelength upwind of the crest (Fig. 2d). The results are averaged over time and over the spanwise direction and they are non-dimensionalized with the free stream velocity U_0. Different lines correspond to the different SGS models under consideration: the Smagorinsky model with two different matching functions (SMAG-1: $C_0 = 0.17$ and $n = 1$; and SMAG-2: $C_0 = 0.1$ and $n = 1$ in Eq. (3)), the Lagrangian dynamic model, and the scale-dependent Lagrangian dynamic model. Results are compared with wind tunnel data (symbols) of Gong et al. (1996). From Fig. 2a we find that the Lagrangian dynamic model clearly overestimates the average velocity near the surface by as much as 20%. This behavior of the velocity profile over the hill crest obtained with the Lagrangian dynamic model is consistent with the velocity overestimation of about 25% reported by Iizuka and Kondo (2004) in their large-eddy simulations of flow over a single two-dimensional hill using the same SGS model. The results from the Smagorinsky model show substantial sensitivity to the choice of parameters and, consequently, the shape of the matching function. The scale-dependent dynamic procedure, which retains the advantage of dynamic models of not requiring any parameter tuning, substantially improves the simulation results with respect to the scale-invariant dynamic model.

The simulated velocity profiles at 1/4 wavelength downwind of the wave crest (Fig. 2b) and in the wave trough (Fig. 2c) show relatively small sensitivity to the SGS model, compared with the results over the wave crest (Fig. 2a). The simulated velocities are close to the measurements above a height of about 30 mm in Fig. 2b and about 50 mm in Fig. 2c. Note that the region below those heights, as reported in Gong et al. (1996), corresponds to the upper limit of a flow recirculation zone that develops downwind of the wave crest. In the recirculation zone, mean velocities are negative and cannot be accurately measured by the hot-wire anemometer, which cannot distinguish between positive and negative velocities and is susceptible to large errors due to flow distortion by the probe support.

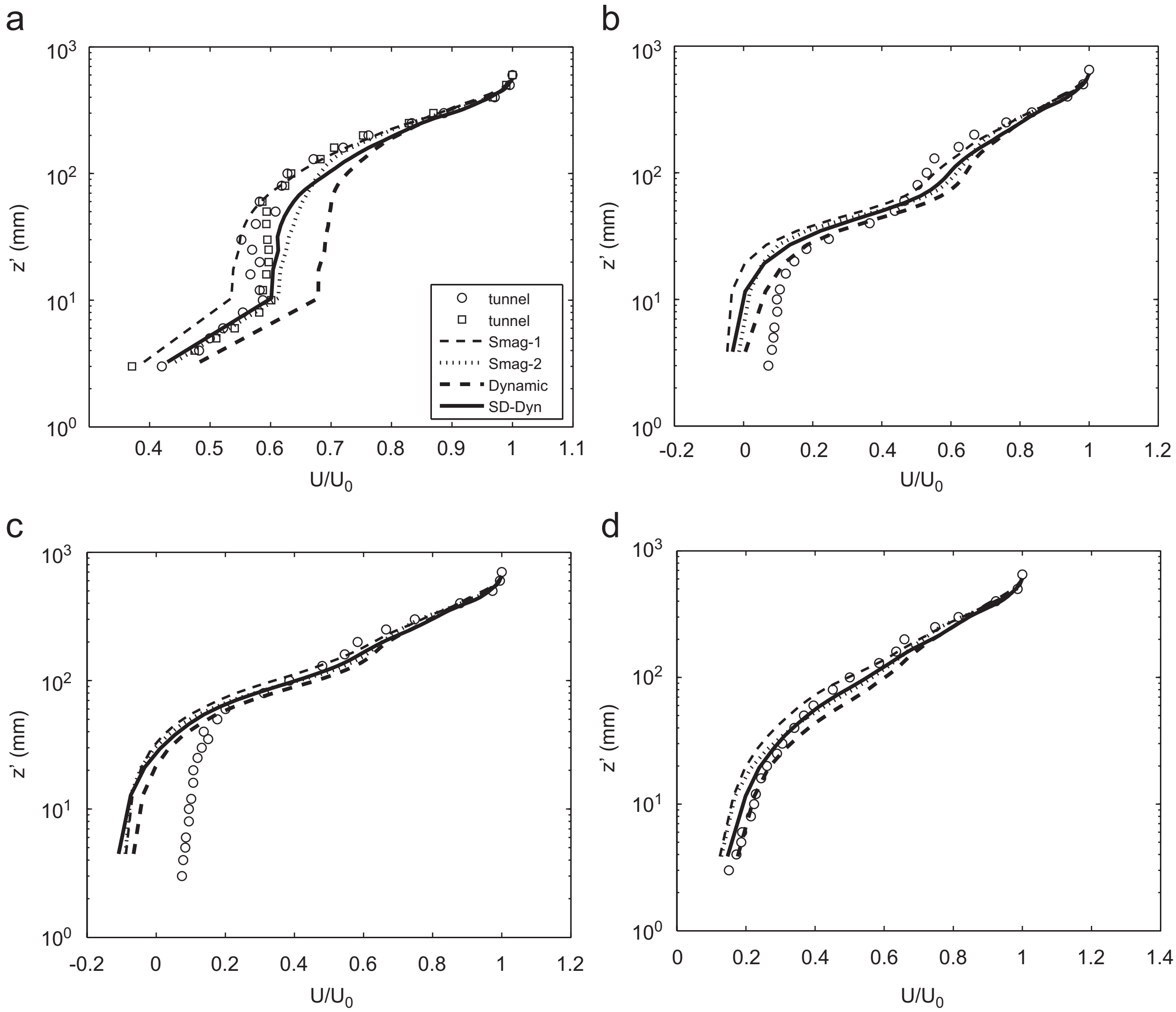

Fig. 2. Non-dimensional velocity profiles from wind tunnel data (symbols) and from LES with different SGS models: Smagorinsky model with two different matching functions (thin dashed and dotted lines), dynamic model (dashed line), and scale-dependent dynamic model (solid line). Results are presented for different positions in the flow: over the wave crests (a); over 1/4 wavelength downwind of the crest (b); over the wave trough (c); and over 1/4 wavelength downwind of the trough (d).

The simulated average velocity 1/4 wavelength upwind of the crest (Fig. 2d) is slightly underestimated by the Smagorinsky model with matching function in the near-surface region, while it is slightly overestimated by the Lagrangian dynamic model at heights between 30 and 200 mm. The scale-dependent Lagrangian dynamic model gives a reasonable prediction throughout most of the domain.

The non-dimensional standard deviations of the resolved streamwise, transverse and vertical velocities over the wave crests are presented in Figs. 3a, 3b and 3c, respectively. Results are again compared with the wind tunnel experimental data (symbols) of Gong et al. (1996). Like in the case of the mean velocity profiles, the standard deviations of the horizontal velocity components simulated with the Smagorinsky model show strong sensitivity to the choice of the matching function for the eddy viscosity coefficient. The Lagrangian dynamic model overpredicts by as much as 50% the level of fluctuations of the horizontal velocity components and also the vertical velocity component. The overestimation of the resolved velocity variance is consistent with the idea that the dynamic model is not dissipative enough, and it is in good agreement

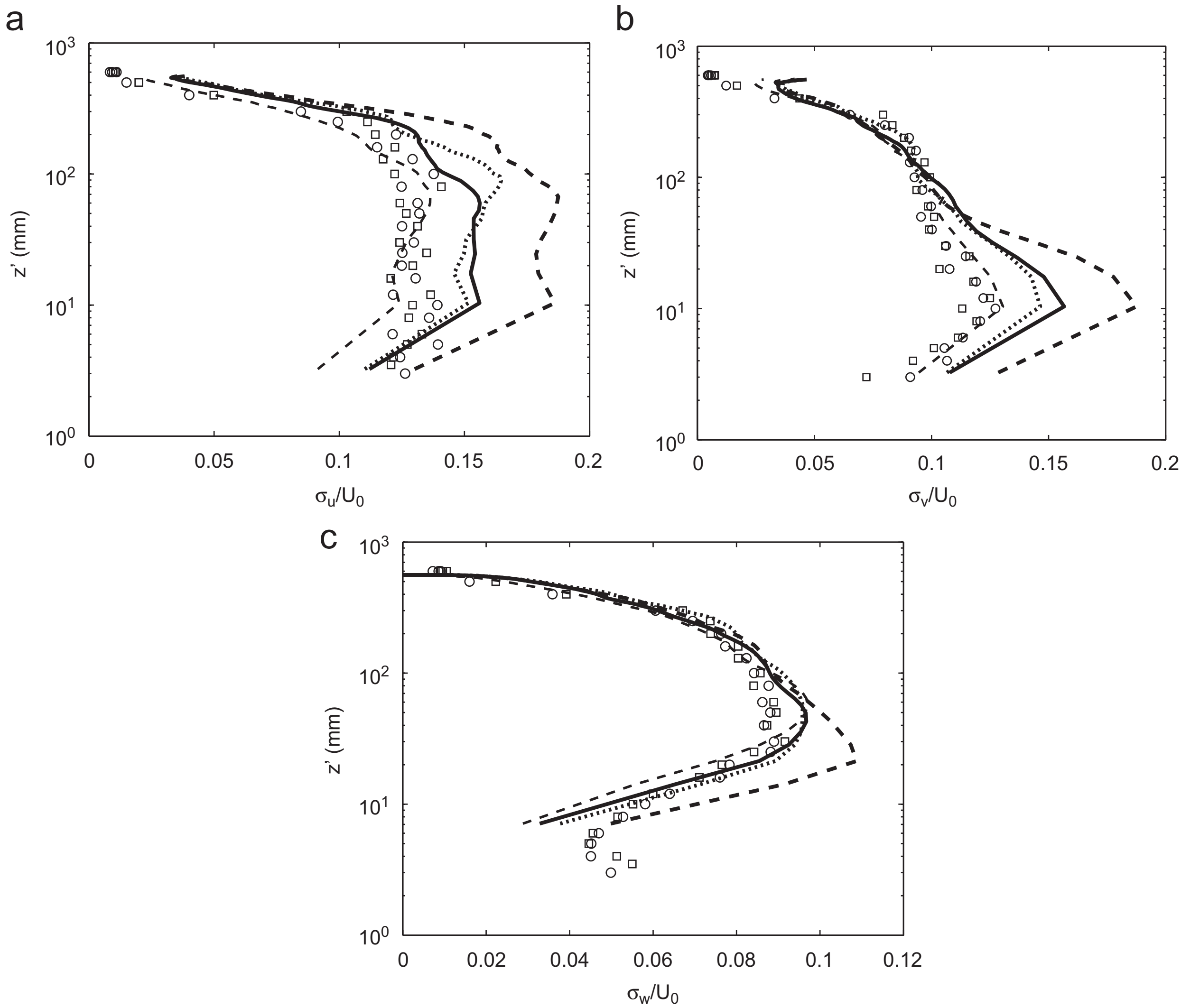

Fig. 3. Non-dimensional standard deviation of the resolved streamwise velocity from wind tunnel data (symbols) and from LES with different SGS models: Smagorinsky model with two different matching functions (thin dashed and dotted lines), dynamic model (dashed line), and scale-dependent dynamic model (solid line). Results are presented for different positions in the flow: over the wave crests (a); over 1/4 wavelength downwind of the crest (b); over the wave trough (c); and over 1/4 wavelength downwind of the trough (d).

with previous studies over flat terrain (Porté-Agel et al., 2000; Bou-Zeid et al., 2005). The Lagrangian scale-dependent dynamic model improves the results with respect to its scale-invariant counterpart, though still overestimating the level of fluctuations of the velocity field.

In order to illustrate the resolution sensitivity of the simulation results, mean velocity profiles over the hill crest, obtained with the scale-invariant and scale-dependent dynamic models, are presented in Figs. 4a and 4b, respectively. The results from the scale-invariant dynamic model show clear resolution dependence, in contrast with the smaller resolution effects obtained with the scale-dependent dynamic model. Similar differences in the resolution effects (not shown here) are found for the simulation results at other locations in the flow.

The dynamically calculated values of the model coefficient C_S^2 obtained using the Lagrangian dynamic and scale-dependent Lagrangian dynamic models are presented in Figs. 5 and 6, respectively. As expected, for any given horizontal position, both coefficients decrease as the distance to the surface decreases in order to account for the reduction in the characteristic scale of the turbulence near the surface. In addition, there is a clear dependence of

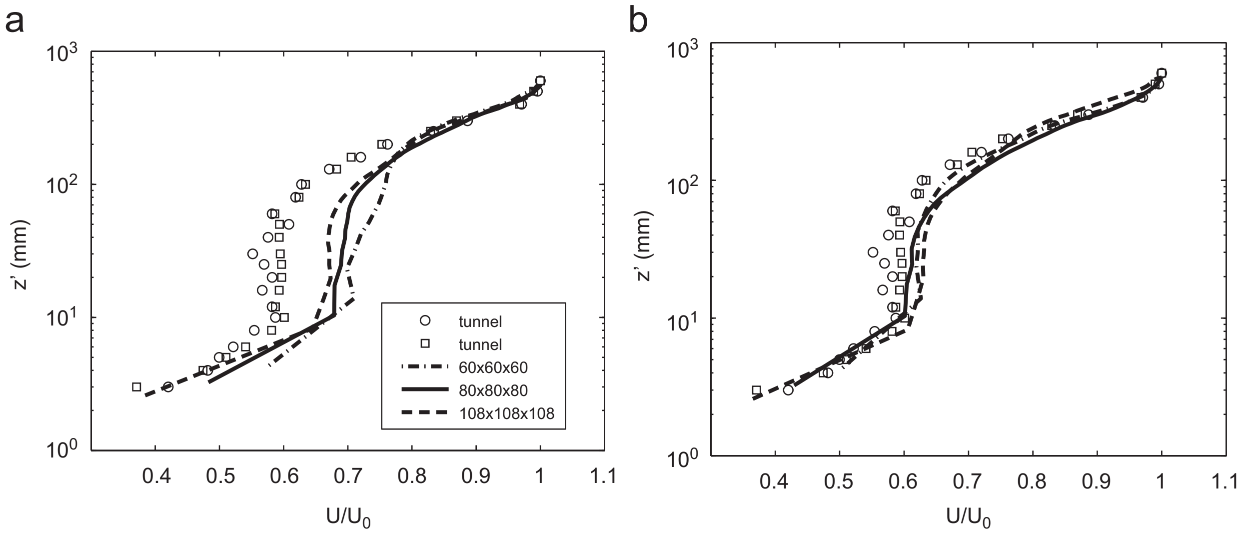

Fig. 4. Effect of grid resolution on the simulated non-dimensional velocity profile over the wave crest obtained with the Lagrangian dynamic model (a) and the scale-dependent Lagrangian dynamic model (b). The wind tunnel data of Gong et al. (1996) (symbols) are also shown.

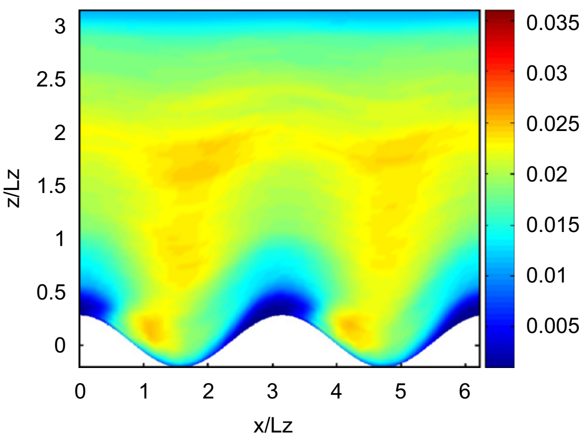

Fig. 5. Smagorinsky coefficient (C_S^2) obtained with the Lagrangian dynamic model. Results are averaged over time and spanwise direction.

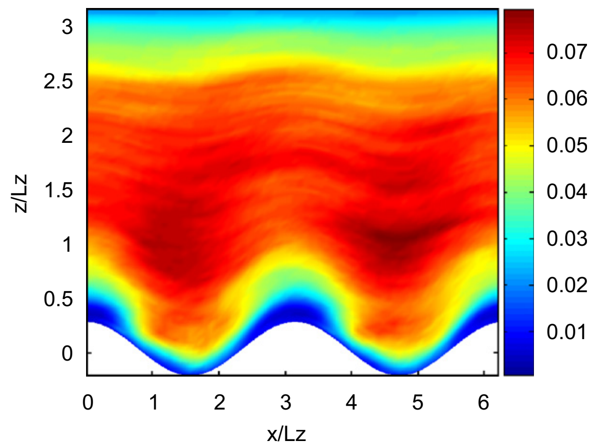

Fig. 6. Smagorinsky coefficient (C_S^2) obtained with the scale-dependent Lagrangian dynamic model. Results are averaged over time and spanwise direction.

the coefficient on horizontal position, associated with the strong non-homogeneity of the flow. For the same distance to the ground, the coefficient is smaller near the crest, where the flow undergoes strong straining. Alternatively, the coefficient is larger in the downwind of the crest, where the flow detaches from the surface (recirculation region) and is subject to smaller strain rates. It is important to note that the value of the coefficient is substantially larger for the scale-dependent dynamic model. The larger value of C_S, together with the increased mean velocity gradients, results in a larger transfer of kinetic energy from the resolved to the sub-grid scales (SGS dissipation). An increase in the rate of removal of energy from the resolved scales leads, in turn, to smaller values of the standard deviations of the resolved velocity, as shown in Fig. 3a–c. Similar trends in the values of the model coefficient, mean velocity and resolved kinetic energy fields were also reported by Porté-Agel et al. (2000) in simulations of a neutral boundary layer using both scale-invariant and scale-dependent dynamic model.

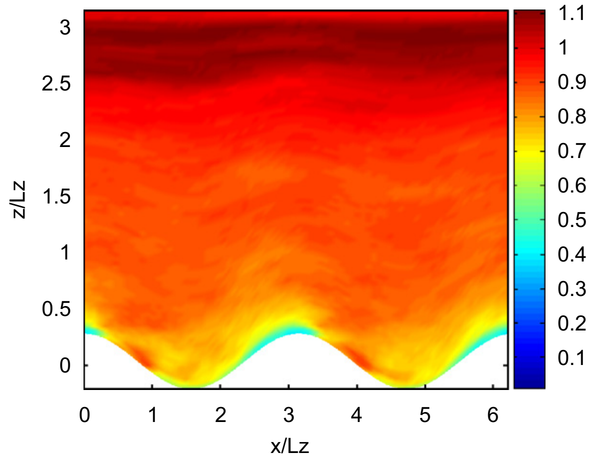

Fig. 7. Scale dependence parameter (β) obtained with the scale-dependent Lagrangian dynamic model. Results are averaged over time and spanwise direction.

Fig. 7 shows the value of the scale dependence parameter β obtained dynamically with the scale-dependent Lagrangian dynamic model. The value of β is close to 1 away from the surface, where the flow is more isotropic at the smallest resolved and subgrid scales and, consequently, C_S^2 is scale invariant. β becomes smaller as the surface is approached due to increased shear and anisotropy of the flow. The smallest values of β are found near the crest, particularly in the upwind side, where the mean shear and anisotropy of the flow are stronger.

4. Summary

Large-eddy simulation (LES) has been used to simulate neutral turbulent boundary-layer flow over a rough two-dimensional sinusoidal hill. Three different subgrid-scale (SGS) models are tested: (a) the standard Smagorinsky model with a wall-matching function, (b) the Lagrangian dynamic model, and (c) the recently developed scale-dependent Lagrangian dynamic model (Stoll and Porté-Agel, 2006). The simulation results obtained with the different models are compared with turbulence statistics obtained from experiments conducted in the meteorological wind tunnel of the Atmospheric Environment Service of Canada (Gong et al., 1996).

The dynamic models have the important advantage of providing tuning-free simulations since the model coefficient is calculated based on the dynamics of the resolved flow. However, the flow simulated using the Lagrangian dynamic model shows important differences compared with the wind tunnel experimental data. In particular, the Lagrangian dynamic model is not dissipative enough, leaving too much kinetic energy in the resolved flow. The model overestimates the magnitude of the velocity over the wave crests by about 20%, which is in agreement with the simulation results of Iizuka and Kondo (2004) in simulations over a single two-dimensional hill.

By relaxing the assumption of scale invariance in the dynamic model, the scale-dependent dynamic model (Porté-Agel et al., 2000; Stoll and Porté-Agel, 2006) is able to dynamically (without any parameter tuning) capture the scale dependence of the model coefficient using information of the smallest resolved scales. Our results show that this procedure substantially improves the simulation results with respect to the scale-invariant dynamic model.

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