Environmental Fluid Dynamics: Lecture 8

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Overview

Basic Equations of Fluid Dynamics

- We are interested in the basic equations of fluids dynamics applied to the atmosphere (i.e., a rotating coordinate system)
- These include
  - Conservation of Mass
  - Conservation of Momentum
  - Conservation of Energy (mechanical and total)
- In general, we would like to determine the transport of mass, momentum, energy, or scalars
- We will present these in differential, Eulerian form

Review:

Lagrangian vs. Eulerian

Review: Lagrangian vs. Eulerian

Lagrangian

- Description of how quantities change with time for an air parcel (following air parcel motion).
- $x(t), y(t), z(t)$ are Cartesian coordinates of the position of the parcel and are dependent variables.
- t is an independent variable.
- $\overrightarrow{r}(t) \equiv x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$ is the parcel position vector.
- $\frac{D()}{DT}$ is the rate of change of $()$.
- $\frac{D}{DT}$ is the (total, substantial, particle, individual, Lagrangian, material) operator.

Review: Lagrangian vs. Eulerian

Lagrangian

- For velocity:

$$\vec{u} \equiv \frac{D\vec{r}}{Dt} = \frac{Dx}{Dt}\hat{i} + \frac{Dy}{Dt}\hat{j} + \frac{Dz}{Dt}\hat{k} = u\hat{i} + v\hat{j} + w\hat{k}$$

- The acceleration is:

$$\vec{a} \equiv \frac{D\vec{u}}{Dt}$$

- Thus,

$$\vec{F} = m\,\vec{a} = m\frac{D\vec{u}}{Dt} = m\frac{D^2\vec{r}}{Dt^2}$$

Review: Lagrangian vs. Eulerian

Eulerian

- Description of how quantities change with time at a fixed point in space (not following parcel).
- $T(x, y, z, t)$ is temperature at a point (x, y, z) in space at time t.
- Here, x, y, z, t are independent variables.
- $\frac{\partial T}{\partial t}$ is the local derivative of T - the time rate of change of T at a fixed point.
- Generally, $D/Dt \neq \partial/\partial t$, but they are related

Relationship Between Lagrangian and Eulerian

- At time t_0, a parcel of air is at x_0, y_0, z_0 with temperature:
  - Lagrangian: $T(t_0)$
  - Eulerian: $T(x_0, y_0, z_0, t_0)$
- Let's consider the parcel at time $t = t_0 + \delta t$.
  - Lagrangian: $T = T(t_0 + \delta t)$
  - Eulerian: $T = T(x_0 + \delta x, y_0 + \delta y, z_0 + \delta z, t_0 + \delta t)$
- Time change in the Lagrangian viewpoint is:

$$\frac{DT}{Dt} = \lim_{\delta t \to 0} \frac{T(t_0 + \delta t) - T(t_0)}{\delta t}$$

- Expand the right-hand side using the Eulerian viewpoint:

$$\frac{DT}{Dt} = \lim_{\delta t \to 0} \frac{T(x_0 + \delta x, y_0 + \delta y, z_0 + \delta z, t_0 + \delta t) - T(x_0, y_0, z_0, t_0)}{\delta t}$$

Relationship Between Lagrangian and Eulerian

- Now we apply the Taylor expansion and neglect higher-order terms.

$$\begin{aligned}\frac{DT}{Dt} &= \lim_{\delta t\to 0}\frac{T_0+\frac{\partial T}{\partial x}\delta x+\frac{\partial T}{\partial y}\delta y+\frac{\partial T}{\partial z}\delta z+\frac{\partial T}{\partial t}\delta t-T_0}{\delta t}\\ &= \lim_{\delta t\to 0}\left(\frac{\partial T}{\partial x}\frac{\delta x}{\delta t}+\frac{\partial T}{\partial y}\frac{\delta y}{\delta t}+\frac{\partial T}{\partial z}\frac{\delta z}{\delta t}+\frac{\partial T}{\partial t}\right)\\ &= \frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\end{aligned}$$

where

$$T_0 = T(x_0, y_0, z_0, t_0)$$

and

$$u=\lim_{\delta t\to 0}\frac{\delta x}{\delta t}, v=\lim_{\delta t\to 0}\frac{\delta y}{\delta t}, w=\lim_{\delta t\to 0}\frac{\delta z}{\delta t}$$

Relationship Between Lagrangian and Eulerian

- So, we have:

$$\underbrace{\frac{DT}{Dt}}_{\text{total deriv}} = \underbrace{\frac{\partial T}{\partial t}}_{\text{local deriv}} + \underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}}_{\text{advection terms}}$$

or

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T$$

- In general,

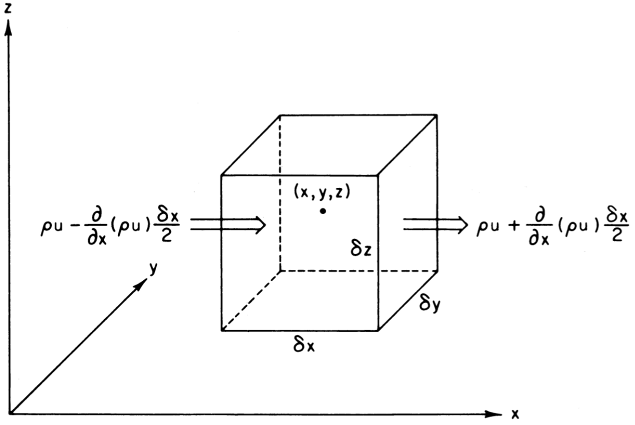
$$\frac{D()}{Dt} = \frac{\partial ()}{\partial t} + \vec{u} \cdot \vec{\nabla} ()$$

Atmospheric Dynamics:

Conservation of Mass

Conservation of Mass

- Consider a stationary volume of fluid through which mass is flowing - conceptually:
  {mass accumulation rate} = {mass in rate} - {mass out rate}



From Holton (2004)

Conservation of Mass

- The rate of mass inflow through RHS

$$\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}$$

- The rate of mass outflow through LHS

$$\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}$$

- Note: area of each face is $\delta y \delta z$, so the net flow into the volume due to u is

$$\left[\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y \delta z - \left[\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y \delta z$$

$$= -\frac{\partial(\rho u)}{\partial x}\delta x \delta y \delta z$$

Conservation of Mass

- Similar expressions hold for the net mass flow into the volume due to v and w, so that

$$\begin{aligned}
\delta x \delta y \delta z \frac{\partial \rho}{\partial t} &= -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \delta x \delta y \delta z \\
\frac{\partial \rho}{\partial t} &= -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \\
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \vec{U})
\end{aligned}$$

  Thus

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) = 0}$$

  the **mass divergence form of the continuity equation**

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) = 0$$

- We can rewrite by using the following relationships

$$\nabla \cdot (\rho \vec{U}) \equiv \rho \nabla \cdot \vec{U} + \vec{U} \cdot \nabla \rho$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{U} \cdot \nabla$$

to arrive at

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{U} + \vec{U} \cdot \nabla \rho = 0$$

$$\boxed{\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{U} = 0}$$

the **velocity divergence form of the continuity equation**

Conservation of Mass

mass divergence form of the continuity equation

- local rate of change of density is balanced by mass divergence

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) = 0$$

velocity divergence form of the continuity equation

- the fractional rate of increase of density following the motion of an air parcel is balanced by the velocity divergence

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{U} = 0$$

Conservation of Mass: Incompressible Flow

- For many cases, the atmosphere may be considered incompressible
- **incompressible flow**: the density of a fluid element does not change during its motion (note: this *does not* imply that density is constant everywhere)

$$\frac{D\rho}{Dt} = 0$$

thus, the continuity equation becomes

$$\boxed{\nabla \cdot \vec{U} = 0}$$

Conservation of Mass: Boussinesq Approximation

- We can extend the assumption of an incompressible flow and take the **Boussinesq approximation**
- Separate the density into 2 parts: a base state $\bar{\rho}(z)$ and perturbation ρ'
- We will apply this after deriving an expression for the momentum balance equation

Atmospheric Dynamics:

Conservation of Momentum

Conservation of Momentum

- Because the Earth's atmosphere has mass, we can apply Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

  where $\vec{F}$ is the sum of all forces acting on an object, m is the mass of the object, and $\vec{a}$ is the acceleration of the object
- In classical mechanics, the object is usually some rigid solid body (*e.g.*, ball, top, pendulum)
- In continuum mechanics, the object is usually some infinitesimal parcel of fluid or an elastic solid
- Since atmospheric dynamics is a branch of continuum mechanics, we will apply Newton's 2nd Law to a small volume element in the atmosphere

Conservation of Momentum: Coordinate System

- We will apply Newton's 2^{nd} Law to a rotating frame of reference. Why? Rotational effects are important for large-scale dynamics in the atmosphere.
- We choose a coordinate system that is fixed to the Earth, which is rotating. Why? That is where we make observations.

Conservation of Momentum: Forces

- There are two categories of forces that we must consider: *fundamental* and *apparent*.
- **Fundamental Forces:** forces directly "felt" by the fluid.
- **Apparent Forces:** imaginary forces that result from acceleration of our coordinate system.

Conservation of Momentum: In Words

- The conservation of momentum may be expressed in words as:

$$
\begin{aligned}
\left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{accumulation}\end{array}\right\} &= \left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{in}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{out}\end{array}\right\}\\
&+ \left\{\begin{array}{c}\text{sum of}\\ \text{fundamental}\\ \text{forces}\end{array}\right\} + \left\{\begin{array}{c}\text{sum of}\\ \text{apparent}\\ \text{forces}\end{array}\right\}
\end{aligned}
$$

Conservation of Momentum: Fundamental Forces

- There are two types of fundamental forces in fluids: *body* and *surface*.
- **Body Forces:** the force on an object is proportional to the mass of the object. This is often referred to as "action at a distance" (*e.g.*, gravity, electromagnetic).
- **Surface Forces:** the force on an object is proportional to the surface area of the object. These forces are due to contact of the object with its surroundings, such as the force on the surface of a fluid element by an outside fluid (*e.g.*, pressure, friction).

Conservation of Momentum: Body Force (Gravity)

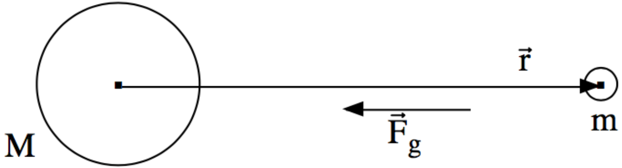
- Let's first derive an expression for the gravitational force, starting with Newton's law of gravitation:

  $$\overrightarrow{F_g} = -\frac{GMm}{r^2}\frac{\overrightarrow{r}}{r}$$

  which applies to two objects with masses M and m, where $\overrightarrow{r}$ is the directed distance between their centers of mass, $r = |\overrightarrow{r}|$, and $G = 6.67 \times 10^{-11}\mathrm{N\ m}\ ^2\mathrm{kg}^{-2}$ is the universal gravitational constant.

Conservation of Momentum: Body Force (Gravity)

- Let $\overrightarrow{r}$ point from a big object of mass M to a little object of mass m. Then $\overrightarrow{F_g}$ is the gravitational force on m due to M.



- Let $\hat{r} \equiv \overrightarrow{r}/r$, so that

$$\overrightarrow{F_g} = -\frac{GMm}{r^2}\hat{r},$$

- Thus, $|\overrightarrow{F_g}|$ is inversely proportional to the square of the distance between the the two masses (if $r \uparrow$ then $|\overrightarrow{F}_g| \downarrow$).

Conservation of Momentum: Body Force (Gravity)

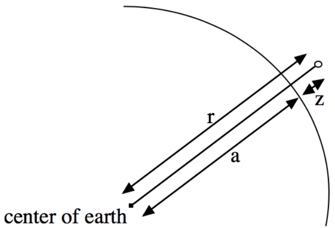
- Let M be the mass of Earth and m be the mass of a small parcel of air. We can then write the gravitational force per unit mass of air as:

$$\underbrace{\frac{\overrightarrow{F_g}}{m}}_{\overrightarrow{g^*}} = -\frac{GM}{r^2}\hat{r}$$

- Consider a parcel of air with mass m at a height z above Earth's surface in the troposphere

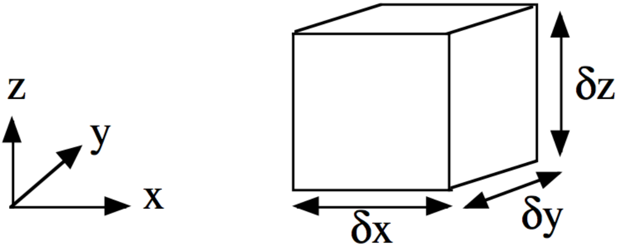
Conservation of Momentum: Body Force (Gravity)

- $a = 6370$ km and $z < 15$ km
- $r = a + z \approx a$, thus $r^2 \approx a^2$
- $\overrightarrow{g^*} \approx -\frac{GM}{a^2}\hat{r}$ (in troposphere)
- $M = \frac{4}{3}\pi a^3 \rho_e$, where $\rho_e = 5520 \text{ kg s}^{-2}$ is the density of Earth
- Thus, $\left|\overrightarrow{g^*}\right| \approx 9.8 \text{ m s}^{-2}$
- This is gravitational acceleration



Conservation of Momentum: Surface Force (Pressure)

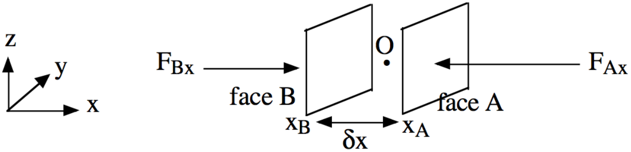
- Consider an infinitesimally small box of air with sides in the x, y, and z directions of length δx, δy, and δz.



- We want to find the net pressure force on the box (note: pressure is a compressive force that acts perpendicular to the surfaces of the box)

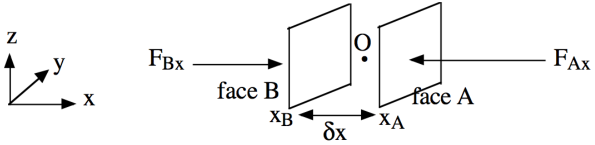
Conservation of Momentum: Surface Force (Pressure)

- Consider the x-component of the pressure force, (only include the two faces of the box perpendicular to the x-axis).



- Point 0 is at the center of the box, located at x_0, y_0, z_0.
- Pressure at center of box is p_0
- F_{Ax} (F_{bx}) is the pressure force on face A (B) in x-direction.
- $X_A = x_0 + \delta x/2$, $X_B = x_0 - \delta x/2$

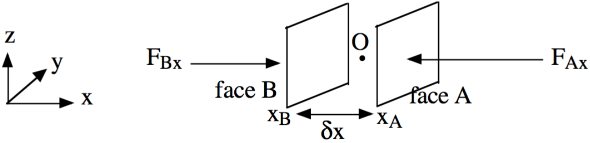
Conservation of Momentum: Surface Force (Pressure)



- Using Taylor series, the pressure on face A is:

$$
\begin{aligned}
p_A &= p_0 + \left.\frac{\partial p}{\partial x}\right|_{x_0, y_0, z_0} (x_A - x_0) + \text{higher order terms (h.o.t.)} \\
&= p_0 + \frac{\partial p}{\partial x}\left(x_0 + \frac{\delta x}{2} - x_0\right) + \cancel{\text{h.o.t.}} \text{ very small} \\
&= p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}
\end{aligned}
$$

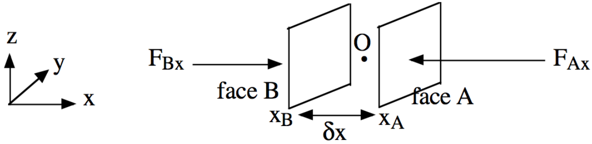
Conservation of Momentum: Surface Force (Pressure)



- Pressure force (pressure × area) on face A is:

$$
\begin{aligned}
F_{Ax} &= -p_A \delta y \delta z \\
&= -\left(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z
\end{aligned}
$$

- Note: the minus sign appears because the force exerted by the outside fluid on the inside fluid across face A is in the minus x-direction.

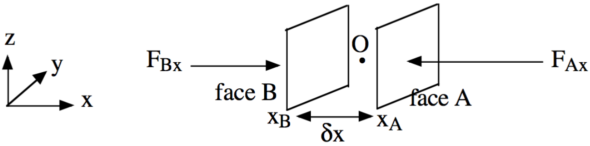


- Using Taylor series, the pressure on face B is:

$$\begin{aligned}
p_B &= p_0 + \left.\frac{\partial p}{\partial x}\right|_{x_0, y_0, z_0} (x_B - x_0) + \text{h.o.t.} \\
&= p_0 + \frac{\partial p}{\partial x}\left(x_0 - \frac{\delta x}{2} - x_0\right) + \cancel{\text{h.o.t.}} \\
&= p_0 - \frac{\partial p}{\partial x}\frac{\delta x}{2}
\end{aligned}$$

very small

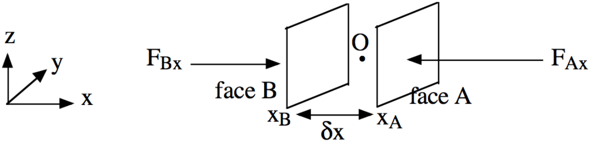
Conservation of Momentum: Surface Force (Pressure)



- Pressure force (pressure*area) on face B is:

$$
\begin{aligned}
F_{Bx} &= p_B \delta y \delta z \\
&= \left(p_0 - \frac{\partial p}{\partial x} \frac{\delta x}{2} \right) \delta y \delta z
\end{aligned}
$$

Conservation of Momentum: Surface Force (Pressure)



- The net x-component pressure force on the box is:

$$\begin{aligned} F_x &\equiv F_{Ax} + F_{Bx} \\ &= -\left(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z + \left(p_0 - \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z \\ &= -\frac{\partial p}{\partial x}\delta x \delta y \delta z \end{aligned}$$

- Thus, the net pressure on the box is proportional to the pressure gradient - the pressure gradient force (PGF).

Conservation of Momentum: Surface Force (Pressure)

- If the mass of the box is $m = \rho \delta V = \rho \delta x \delta y \delta z$, then the x-component PGF per unit mass is:

$$\frac{F_x}{m} = -\frac{\dfrac{\partial p}{\partial x} \delta x \delta y \delta z}{\rho \delta x \delta y \delta z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Similarly,

$$\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \text{and} \qquad \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

- In vector form:

$$\boxed{\frac{\overrightarrow{F_p}}{m} = -\frac{1}{\rho}\vec{\nabla} p}$$

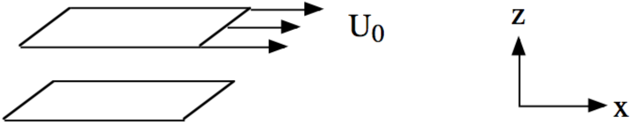
Conservation of Momentum: Surface Force (Pressure)

$$\boxed{\frac{\overrightarrow{F_p}}{m} = -\frac{1}{\rho}\vec{\nabla} p}$$

- Note: The PGF is proportional to the gradient of pressure and not the pressure itself.
- The magnitude of the PGF is large where the magnitude of $\vec{\nabla} p$ is large (tight isobars).
- The leading minus sign indicates that the PGF acts in the opposite direction of $\vec{\nabla} p$ (from high to low pressure).

Conservation of Momentum: Surface Force (Friction)

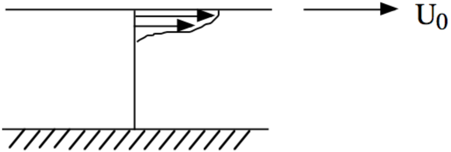
- Now we consider the surface force due to molecular friction - the viscous force.
- To understand this force, consider fluid at rest between two infinite parallel plates. At time $t = 0$ the top plate begins moving at a speed of U_0 in the x direction:



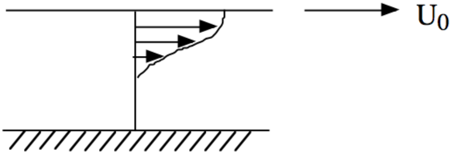
- The "no-slip" condition means that molecular friction causes fluid to stick to solid objects or boundaries.

- Fluid at the lower plate does not moves because it is sticking to the non-moving boundary.
- Fluid at the top plate moves at speed U_0 because it is sticking to the moving boundary.

t = 1 min:

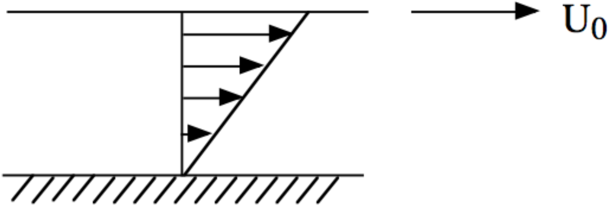


t = 2 min:



Conservation of Momentum: Surface Force (Friction)

- Eventually we get a steady-state, where there is no change in velocity with time at any point.
- For this experiment, steady-state looks like:



- We find that $u = U_0 \frac{z}{h}$, where h is the distance between the two plates.
- From this, we see that $\partial u / \partial z = U_0 / h$.
- Thus, we get a linear profile.

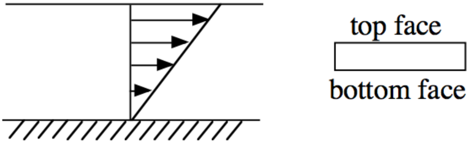
Conservation of Momentum: Surface Force (Friction)

- Why do we get a linear profile of velocity when this experiment reaches steady-state?
- Consider the x-component of the viscous force, per unit area, exerted on a horizontal area at height z by the overlying fluid:

  $$\tau_{zx} = \mu \frac{\partial u}{\partial z}$$

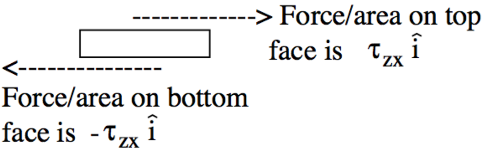
  where the subscripts denote that we are considering the force in the x direction at height z, and μ is the dynamic viscosity.
- τ_{zx} is the shearing stress and represents one component of the stress tensor.
- This shearing stress is proportional to the vertical gradient of the x-component of velocity.

- Consider the x-component force balance on a parcel of fluid within the flow:



- Top face: $\partial u/\partial z > 0 \rightarrow \tau_{zx} > 0 \rightarrow$ the x-component force exerted on the bottom face by the overlying fluid is positive.
- The underlying fluid exerts an equal and opposite force on the bottom face via Newton's 3rd Law (action/reaction).
- This means that the slow fluid underlying the bottom face tries to slow down the faster fluid above it in the parcel.

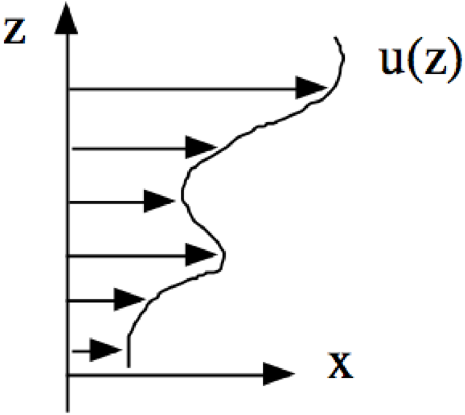
Conservation of Momentum: Surface Force (Friction)



- Because u varies linearly with height, $\partial u/\partial z$ is spatially constant.
- This means that τ_{zx} is constant $\rightarrow$ the forces on the top and bottom faces are equal and opposite.
- The result is that there is no net horizontal force, and thus no horizontal acceleration.
- This implies steady-state for this experiment.

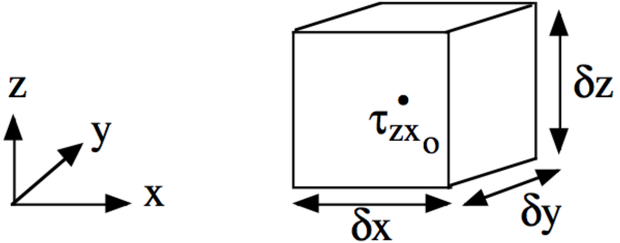
Conservation of Momentum: Surface Force (Friction)

- Let's move beyond the simple plate experiment and consider a unidirectional shear flow in which $u(z)$ is not linear.
- In this case, τ_{zx} is not spatially constant $\rightarrow$ no steady-state.



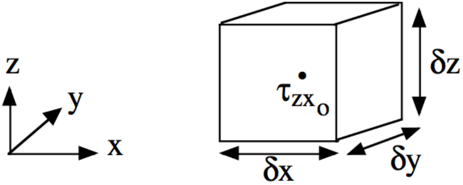
Conservation of Momentum: Surface Force (Friction)

- Consider an infinitesimally small box of fluid with sides in the x, y, and z directions of length δx, δy, and δz.



- τ_{zx0} is the x-component of shearing stress at the center of the box.

Conservation of Momentum: Surface Force (Friction)



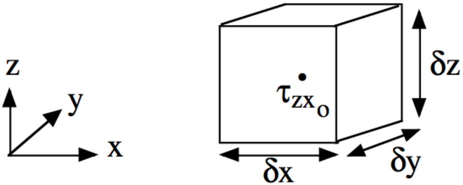
- The x-component of shearing stress at the top of the box is:

$$\tau_{zx0} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}$$

- The x-component viscous force on the top face exerted by the overlying fluid is:

$$\left(\tau_{zx0} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta x \delta y$$

Conservation of Momentum: Surface Force (Friction)



- The x-component of shearing stress at the bottom of the box is:

$$\tau_{zx0} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}$$

- The x-component viscous force on the bottom face exerted by the underlying fluid is:

$$-\left(\tau_{zx0} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}\right)\delta x \delta y$$

Conservation of Momentum: Surface Force (Friction)

- The total x-component viscous force on the fluid box is:

$$\overrightarrow{F_f} = \left(\tau_{zx0} + \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}\right)\delta x \delta y - \left(\tau_{zx0} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}\right)\delta x \delta y$$
$$= \frac{\partial \tau_{zx}}{\partial z}\delta x \delta y \delta z$$

The total x-component viscous force per unit mass is:

$$F_x = \frac{\frac{\partial \tau_{zx}}{\partial z}\delta x \delta y \delta z}{\rho \delta x \delta y \delta z} = \frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \text{ assume } \mu\text{=contant}$$
$$= \frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}$$
$$= \nu \frac{\partial^2 u}{\partial z^2}$$

where $\nu = \mu/\rho$ is kinematic viscosity.

Conservation of Momentum: Surface Force (Friction)

- In reality, u will vary in the x, y, and z directions $\rightarrow$ $u = u(x, y, z)$. Thus,

$$\frac{F_x}{m} = \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \nu \vec{\nabla}^2 u$$

In general, there are also $v = v(x, y, z)$ and $w = w(x, y, z)$ components to consider:

$$\frac{F_y}{m} = \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \nu \vec{\nabla}^2 v$$

$$\frac{F_z}{m} = \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \nu \vec{\nabla}^2 w$$

or in vector form:

$$\frac{\vec{F_f}}{m} = \boxed{\nu \left(\frac{\partial^2 \vec{u}}{\partial x^2} + \frac{\partial^2 \vec{u}}{\partial y^2} + \frac{\partial^2 \vec{u}}{\partial z^2} \right) = \nu \vec{\nabla}^2 \vec{u}}$$

Conservation of Momentum: Surface Force (Friction)

- The physical basis for viscous force in the atmosphere is the random migration of air molecules.
- Consider a large-scale flow where u increases with height.
- Downward-moving molecules have larger u than upward-moving molecules.
- Thus, faster momentum is brought down and slower momentum is moved upward (mixing) $\rightarrow$ bigger $\partial u/\partial z$ means greater momentum transport by frictional effects.
- Is the net effect dominated by downward or upward momentum transport? This is determined by the vertical derivative ($\partial/\partial z$) of the shear ($\partial u/\partial z$).

Conservation of Momentum: Surface Force (Friction)

- The origin of friction in liquids is much more complicated.
- There is some attraction between molecules, but not much in the way of migration.
- Notably, we get the same mathematical description of the viscous force as for gases ($\overrightarrow{F_f} = \nu \vec{\nabla}^2 \vec{u}$) - just with a different value for ν.
- However, for $T \uparrow$ we get that $\nu_{\text{water}} \downarrow$ and $\nu_{\text{air}} \uparrow$.