Environmental Fluid Dynamics

ME EN 7710 | Homework #2 | Due: March 24th

1.) Boundary Layer Profiles

Given the provided ballon data (http://gibbs.science/efd/homework/balloon_data.txt), perform the following:

(a) Plot vertical profiles of all the raw variables on a single page with clearly labeled axes. Note any interesting features.

(b) Calculate and plot the (i) temperature, (ii) potential temperature, and (iii) virtual potential temperature. Compare and discuss the potential temperature and virtual potential temperatures. Why do they differ?

(c) Compare the potential temperature calculations using the balloon-based pressure measurements with $\theta(z) \approxeq T(z) + \Gamma z$. Calculate and plot the difference. What is the difference?

2.) The Laminar Ekman Layer Above a Rigid Surface

The simplified momentum equations:

$$-fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}$$
$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2}$$

can be further simplified by expressing pressure gradients in terms of geostrophic velocity components as:

$$-f(v - V_g) = \nu\frac{\partial^2}{\partial z^2}(u - U_g) \tag{1}$$
$$f(u - U_g) = \nu\frac{\partial^2}{\partial z^2}(v - V_g) \tag{2}$$

(a) Assuming U_g and V_g are height independent, solve Eqs. (1) and (2) subject to the following boundary conditions:

- $u(z{=}0) = v(z{=}0) = 0$
- $u(z \to \infty) = U_g$, $v(z \to \infty) = V_g$

For the final solution, orient the x-axis with the geostrophic wind vector (i.e., U_g = U and V_g = 0). The solutions should be in the form (where a is the inverse Ekman depth):

$$u = U[1 - e^{-az}\cos(az)] \qquad v = Ge^{-az}\sin(az)$$

(b) Plot your solution as a hodograph and as vertical profiles of u and v.

(c) Please explain the phenomena of Ekman pumping.