Environmental Fluid Dynamics

ME EN 7710 Homework #1 Solutions

1.) Arya - Chapter 2, Exercise 5 (4 pts)

Explain the following terms or concepts used in connection with the surface energy budget:

(a) "ideal" surface
A surface that is infinitely thin (no mass or heat capacity), very wide and horizontally homogeneous, and opaque to radiation

(b) evaporative cooling
Cooling as a result of phase change (liquid to gas in this case). That is, energy is required to supply the latent heat of vaporization

(c) oasis effect
Cooling associated with latent heat flux when warm dry air is advected over a moist surface. Significant evaporation results in cooling of the surface, which results in a downward sensible heat flux

(d) flux divergence
When the energy input into a layer of the atmosphere is less than the energy leaving the layer

2.) Arya - Chapter 3, Exercise 2 (8 pts)

(a) Estimate the combined sensible and latent heat fluxes from the surface to the atmosphere, given the following observations:

- Incoming shortwave radiation = 800 W m^{-2}
- Heat flux to the submedium = 150 W m^{-2}
- Albedo of the surface = 0.35

Assume ideal surface ($\Delta H_s = 0$) and the budget is given as $R_N = H_S + H_L + H_G$, where $R_N = R_{S\downarrow} + R_{S\uparrow} + R_{L\downarrow} + R_{L\uparrow}$. Assume that $R_{L\downarrow} \approx R_{L\uparrow}$ and write $R_{S\uparrow} = -\alpha R_{S\downarrow}$. Rearrange to get

$$\begin{aligned} H_S + H_L &= R_{S\downarrow}(1-\alpha) - H_G \\ &= 800 \text{ W m}^{-2}(1-0.35) - 150 \text{ W m}^{-2} \\ &\boxed{= 370 \text{ W m}^{-2}} \end{aligned}$$

(b) What would be the result if the surface albedo were to drop to 0.07 after irrigation?

$$\begin{aligned} H_S + H_L &= 800 \text{ W m}^{-2}(1-0.07) - 150 \text{ W m}^{-2} \\ &\boxed{= 594 \text{ W m}^{-2}} \end{aligned}$$

3.) Arya - Chapter 3, Exercise 3 (12 pts)

The following measurements or estimates were made of the radiative fluxes over a short grass surface during a clear sunny day:

- Incoming shortwave radiation: 675 W m^{-2}
- Incoming longwave radiation: 390 W m^{-2}
- Ground surface temperature: 35°C
- Albedo of the surface: 0.20
- Emissivity of the surface: 0.92

(a) From the radiation balance equation, calculate the net radiation at the surface.

$$\begin{aligned} R_N &= R_{S\downarrow} + R_{S\uparrow} + R_{L\downarrow} + R_{L\uparrow} \\ &= R_{S\downarrow}(1-\alpha) + R_{L\downarrow} - \epsilon\sigma T_S^4 \\ &= 675 \text{ W m}^{-2}(1-0.2) + 390 \text{ W m}^{-2} - (0.92)(5.67\times10^{-8} \text{ W m}^{-2}\text{ K}^{-4})(308 \text{ K})^4 \\ &\boxed{= 460.6 \text{ W m}^{-2}} \end{aligned}$$

(b) What would be the net radiation after the surface is thoroughly watered so that its albedo drops to 0.10 and its effective surface temperature reduces to 25°C?

$$\begin{aligned} R_N &= 675 \text{ W m}^{-2}(1-0.10) + 390 \text{ W m}^{-2} - (0.92)(5.67\times10^{-8} \text{ W m}^{-2}\text{ K}^{-4})(298 \text{ K})^4 \\ &\boxed{= 586.1 \text{ W m}^{-2}} \end{aligned}$$

(c) Qualitatively discuss the effect of watering on the other energy fluxes to or from the surface.
We will expect an increased latent heat flux and potentially a decreased sensible heat flux and ground heat flux as the surface is cooled.

4.) Arya - Chapter 3, Exercise 7 (6 pts)

Discuss the merits of the proposition that net radiation R_N can be deduced from measurements from solar radiation $R_{S\downarrow}$ during the daylight hours, using the empirical relationship

$$R_N = AR_{s\downarrow} + B$$

where A and B are constants. On what factors are A and B expected to depend?

$$\begin{aligned} &R_N = AR_{s\downarrow} + B \\ &\quad A \sim (1-\alpha) \Rightarrow \text{depends on albedo} \\ &\quad B \sim R_L \text{or net longwave radiation} \Rightarrow \text{this is roughly constant and small during day} \end{aligned}$$

5.) Boltzmann and Planck (10 pts)

Derive Stefan-Boltzmann's Law from Planck's Law.

Planck's Law

$$R_\lambda = \frac{C_1}{\lambda^5}\frac{1}{[\exp(C_2/\lambda T) - 1]}$$

Integrate

$$\begin{aligned} R &= \int_0^\infty R_\lambda d\lambda \\ &= \int_0^\infty \frac{C_1}{\lambda^5}\frac{1}{[\exp(C_2/\lambda T) - 1]} d\lambda \\ &= C_1 \int_0^\infty \frac{1}{\lambda^5}\frac{1}{[\exp(C_2/\lambda T) - 1]} d\lambda \end{aligned}$$

$$\begin{aligned} u &= \frac{C_2}{\lambda T} \\ \lambda &= \frac{C_2}{uT} \\ \frac{d\lambda}{du} &= -\frac{1}{u^2}\frac{C_2}{T} \Rightarrow d\lambda = -\frac{C_2}{T}\frac{1}{u^2} du \end{aligned}$$

$$\begin{aligned} R &= C_1 \int_\infty^0 \frac{1}{\left(\frac{C_2}{uT}\right)^5} \frac{-\frac{C_2}{T}\frac{1}{u^2}}{[\exp(C_2/\lambda T) - 1]} du \\ &= C_1 \int_\infty^0 \left(\frac{T}{C_2}\right)^4 \frac{-u^3}{e^u - 1} du \\ &= C_1 \left(\frac{T}{C_2}\right)^4 \underbrace{\int_0^\infty \frac{u^3}{e^u - 1} du}_{=\pi^4/15 \text{ via integral tables}} \\ &= \frac{C_1}{15}\left(\frac{\pi}{C_2}\right)^4 T^4 \\ &\boxed{= \sigma T^4} \rightarrow \text{ Stefan-Boltzmann Law} \end{aligned}$$

where

$$\begin{aligned} C_1 &= 3.742 \times 10^8 \text{ W } \mu\text{m}^4 \text{ m}^{-2} \\ C_2 &= 1.439 \times 10^4 \text{ } \mu\text{m K} \\ \sigma &= \frac{C_1}{15}\left(\frac{\pi}{C_2}\right)^4 = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \end{aligned}$$

6.) Wein and Planck (10 pts)

Derive Wein's Law from Planck's Law.

$$R_\lambda = \frac{C_1}{\lambda^5}\frac{1}{[\exp(C_2/\lambda T) - 1]}$$

$$\begin{aligned}\frac{dR_\lambda}{d\lambda} &= C_1\left(\frac{C_2}{T\lambda^7}\frac{\exp(C_2/\lambda T)}{[\exp(C_2/\lambda T) - 1]^2} - \frac{1}{\lambda^6}\frac{5}{\exp(C_2/\lambda T) - 1}\right) = 0\\ &= \frac{C_2}{\lambda T}\frac{\exp(C_2/\lambda T)}{\exp(C_2/\lambda T) - 1} - 5 = 0\end{aligned}$$

define

$$u \equiv \frac{C_2}{\lambda T}$$

and the equation becomes

$$u\frac{\exp(u)}{\exp(u) - 1} - 5 = 0$$

This can be solved numerically to give $u = 4.965$, and thus

$$\lambda_{\text{max}} = \frac{C_2}{uT} = \boxed{\frac{2898\ \mu\text{m K}}{T}} \to \text{Wein's Displacement Law}$$

where $C_2 = 1.439 \times 10^4\ \mu\text{m K}$