A REVIEW OF FLUX-PROFILE RELATIONSHIPS

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Abstract. Flux-profile relationships in the constant flux layer are reviewed. The preferred relationships are found to be those of Dyer and Hicks (1970), namely, $\phi_H = \phi_W = (1 - 16(z/L))^{-1/2}$, $\phi_M = (1 - 16(z/L))^{-1/4}$ for the unstable region, and $\phi_H = \phi_W = \phi_M = 1 + 5(z/L)$ for the stable region.
The carefully determined results of Businger *et al.* (1971) remain a difficulty which calls for considerable clarification.

1. Introduction

The long-standing interest in flux-profile relationships stems from two main sources. Firstly, the recognition that a knowledge of the eddy fluxes of heat, water vapour and momentum close to the surface is essential to a proper understanding of the workings of the atmosphere, and secondly, the important need in numerical simulation to have a means of estimating these fluxes from explicit parameters occurring in the model. The second of these is still not satisfactorily resolved since the relatively coarse grids of atmospheric modelling have had the effect of shifting the emphasis from a constant flux layer description to a boundary-layer description.

There was, also, in the early days of the subject, the purely pragmatic aspect that whilst wind profiles could be obtained easily, temperature profiles fairly readily, and humidity profiles with some difficulty, there was no means at all (with the exception of early lysimeters and drag plates) of direct measurement of the eddy fluxes.

The situation has now changed considerably. The development of the eddy-correlation technique, to which Swinbank made a notable pioneering contribution, has now reached a stage where the direct measurement of eddy fluxes can be achieved with the same ease as profiles.

There is still, however, a basic scientific need to understand fully the relationships between fluxes and profiles. The literature of the last twenty years is liberally scattered with suggestions, both theoretical and experimental. Whilst the theoretical work has laid a fairly firm foundation for the interpretation of the experimental material in dimensionless terms, there is still no fundamental theory capable of predicting the required relationships. On the experimental side, there is still no universal acceptance of the observational material.

This paper is written in an attempt to review and comment on the present position.

2. Mathematical Background

It is now common practice to discuss flux-profile relationships in terms of the stability parameter z/L. Earlier descriptions used the Richardson Number, Ri, in a similar

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fashion. Use was also made of the transfer coefficients K_H, K_W and K_M for heat, water and momentum, respectively. For the sake of clarity, the following relations are set down with the usual notation.

$$L = -\frac{\rho C_p u_*^3 \theta}{kgH}$$

$$\mathrm{Ri} = \frac{g}{\theta} \frac{\dfrac{\partial \theta}{\partial z}}{\left(\dfrac{\partial u}{\partial z}\right)^2}$$

$$H = -\rho C_p K_H \frac{\partial \theta}{\partial z}$$

$$E = -\rho L_W K_W \frac{\partial q}{\partial z}$$

$$\tau = \rho K_M \frac{\partial u}{\partial z}.$$

Largely on the basis of a dimensional argument, Monin and Obukhov (1954) established the following flux-profile generalisations:

$$\frac{\partial \theta}{\partial z} = -\frac{H}{\rho C_p k u_* z}\, \phi_H (z/L)$$

$$\frac{\partial q}{\partial z} = -\frac{E}{\rho L_W k u_* z}\, \phi_W (z/L)$$

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}\, \phi_M (z/L).$$

The specification of the required flux-profile relationships reduces therefore to a knowledge of the universal functions ϕ_H, ϕ_W and ϕ_M as a function of z/L.

It follows directly from the foregoing that

$$K_H = ku_* z/\phi_H$$
$$K_W = ku_* z/\phi_W$$
$$K_M = ku_* z/\phi_M$$

and

$$\mathrm{Ri} = \frac{z}{L} \frac{\phi_H}{\phi_M^2}.$$

Thus, once the functions ϕ_M and ϕ_H are known as a function of z/L, the relationship between Ri and z/L is established. Similarly the variation of K_H/K_M with stability is also determined.

It is also necessary, in the most complete formulation, to take due account of the buoyancy effect of the water vapour flux. This is usually done by replacing H in the expression for L by $(H+0.07E)$, and the potential temperature term $\partial\theta/\partial z$ in the expression for Ri with the virtual potential temperature gradient $(\partial\theta_V/\partial z)$. The relationships between Ri, K_H/K_M and z/L therefore require the appropriate manipulation of three equations involving ϕ_M, ϕ_H and ϕ_W rather than two as set out above. In most cases it is not necessary to consider this degree of refinement, though notable exceptions occur when $H \ll E$ which is not unusual over the oceans. Most readers will be aware of the implications of the more complete description, which in the interests of simplicity, will not be used in the following.

It is clear that, in near-neutral conditions, ϕ_M will approach unity asymptotically, thus yielding the long-established logarithmic wind profile. Following the usual, although not universally accepted assumption, that $K_H = K_M = K_W$ under neutral conditions, ϕ_H and ϕ_W will also approach unity asymptotically. It is often assumed on a not very firm foundation that the asymptotic approach of ϕ_H and ϕ_W to unity will be identical with that for ϕ_M, i.e., that the coefficient α in the log-linear form $\phi = 1 + \alpha(z/L)$ is the same for all three ϕ functions. There is now strong evidence that this is not so in unstable conditions.

Some recent evidence, to be discussed later, argues that K_H/K_M at neutral stability is equal to 1.35, in conflict with the more generally accepted situation of equality. A similarly disturbing suggestion from the same source is that the von Karman constant k is equal to 0.35 rather than the value of 0.41 generally accepted previously. These matters will be commented on more fully in the following.

3. Comparison of Flux-Profile Relationships

A thorough assessment of all the flux-profile relationships that have appeared in the literature would be an enormous, if not impossible task, particularly as some of the experimental observations are of less value than others. The subject is further clouded by the fact that some of the relationships are expressed in terms of Ri, usually because direct measurements of the eddy fluxes were not available, so that the dependence upon z/L must presume an Ri$-z/L$ relationship which can usually not be assessed from the data.

This difficulty can be illustrated by one example, which is not intended to be overcritical. In describing the KEYPS profile, for unstable stratification, Lumley and Panofsky (1964) consider that both wind and temperature profiles can be described by a unified equation

$$\phi^4 - \gamma \frac{z}{L} \phi^3 = 1,$$

where γ is a constant. But since heat flux observations were not available, Panofsky transforms this equation to

$$\phi^4 - \gamma' \frac{z}{L'} \phi^3 = 1,$$

TABLE I

A summary of ϕ forms recently suggested

Authors	Comments	Unstable		
		Heat	Water Vapour	Momentum
Swinbank (1964)	Exponential wind profile	–	–	$\phi_M = (z/L)\,[1 - \exp(z/L)]^{-1}$
Swinbank (1968)	Using $k = 0.40$, u_* observed from drag coefficient $-0.1 > z/L > -2$	$\phi_H = 0.227(-z/L)^{-0.44}$	–	$\phi_M = 0.613(-z/L)^{-0.20}$
Webb (1970)	Profiles only, no direct eddy fluxes $z/L > -0.03$	$\phi_H = 1 + 4.5(z/L)$	$\phi_W = 1 + 4.5(z/L)$	$\phi_M = 1 + 4.5(z/L)$
Dyer and Hicks (1970)	Using $k = 0.41$, comparison of direct eddy fluxes and profiles $(0 > z/L > -1)$	$\phi_H = (1 - 16(z/L))^{-1/2}$	$\phi_W = (1 - 16(z/L))^{-1/2}$	$\phi_M = (1 - 16(z/L))^{-1/4}$
Businger *et al.* (1971)	Using $k = 0.35$, comparison of direct eddy fluxes and profiles $(z/L > -2)$	$\phi_H = 0.74(1 - 9(z/L))^{-1/2}$	–	$\phi_M = (1 - 15(z/L))^{-1/4}$

Table I (continued)

Authors	Comments	Stable		
		Heat	Water vapour	Momentum
Swinbank (1964)	Exponential wind profile	–	–	–
Swinbank (1968)	Using $k = 0.40$, u_* observed from drag coefficient $-0.1 > z/L > -2$	–	–	–
Webb (1970)	Profiles only, no direct eddy fluxes, $z/L > -0.03$	$\phi_H = 1 + 5.2(z/L)$	$\phi_N = 1 + 5.2(z/L)$	$\phi_M = 1 + 5.2(z/L)$
Dyer and Hicks (1970)	Using $k = 0.41$, comparison of direct eddy fluxes and profiles $(0 > z/L > -1)$	–	–	–
Businger *et al.* (1971)	Using $k = 0.35$, comparison of direct eddy fluxes and profiles $(z/L > -2)$	$\phi_H = 0.74 + 4.7(z/L)$	–	$\phi_M = 1 + 4.7(z/L)$

where $\gamma' = \gamma K_H/K_M$. The assumption that γ' is constant implies K_H/K_M is also constant (about 1.3), thus apparently verifying the unifying concept of the earlier equation, but obviously prejudging an important question. Panofsky comments, quite correctly, that this matter is highly controversial.

In view of these sorts of difficulties in earlier presentations, this author has chosen to discuss only those of the more recent ones which he believes contain measurements of sufficient quality to merit consideration. Table I has been constructed on this basis and the material is presented in graphical form in Figures 1, 2 and 3. All of the formulae in Table I are empirical expressions, with the exception of the Swinbank exponential wind profile which is a mathematical statement emerging from a single hypothesis together with acceptance of the neutral logarithmic wind profile.

Two recent reviews should be read in conjunction with the present paper. Busch (1973) discusses some of the material used here, and also refers to some other recent work. Monin and Yaglom (1971) provide a very comprehensive review of the subject, both theoretical and experimental. One of the intentions of the present paper is to offer some comments that have not been made previously.

The results of Dyer and Hicks (1970) and Businger *et al.* (1971) probably represent the most satisfactory experiments for the unstable case in that eddy fluxes and vertical profiles were independently determined and then related within the z/L framework.

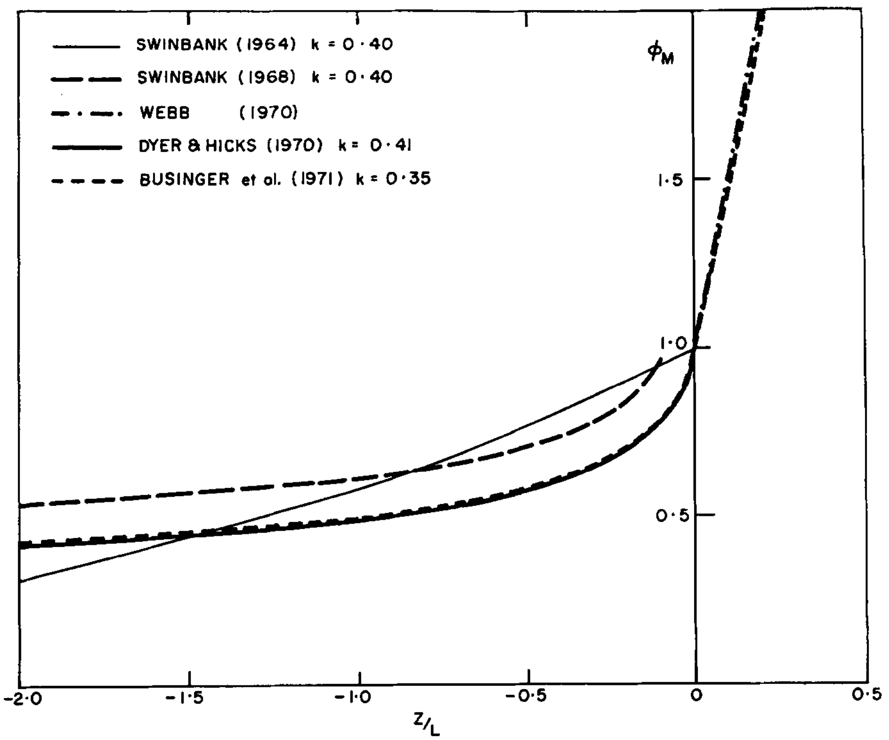

Fig. 1. ϕ_M plotted as a function of z/L.

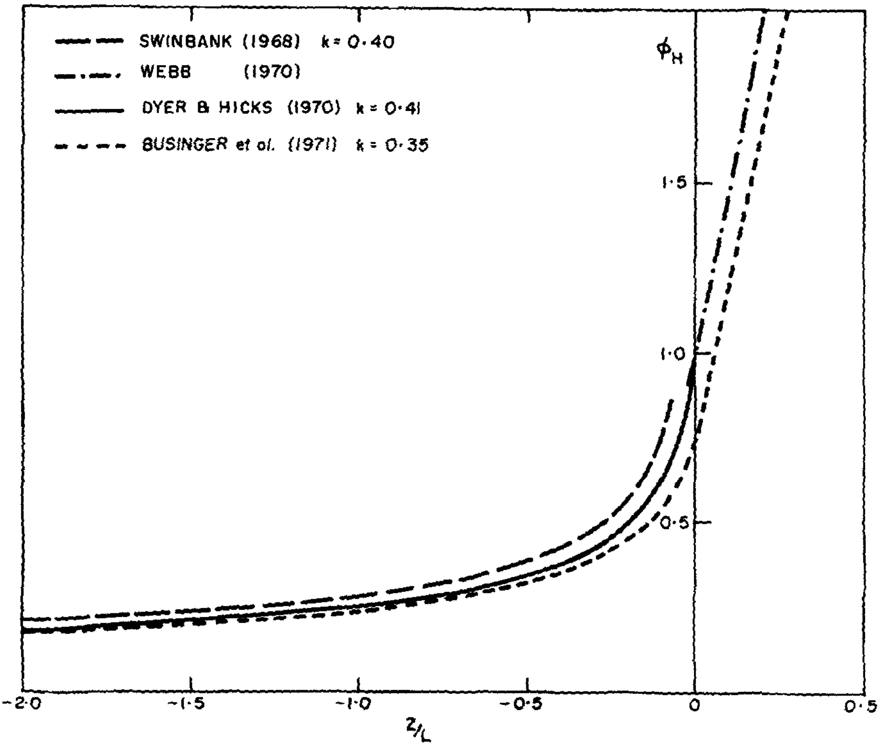

Fig. 2. ϕ_H plotted as a function of z/L.

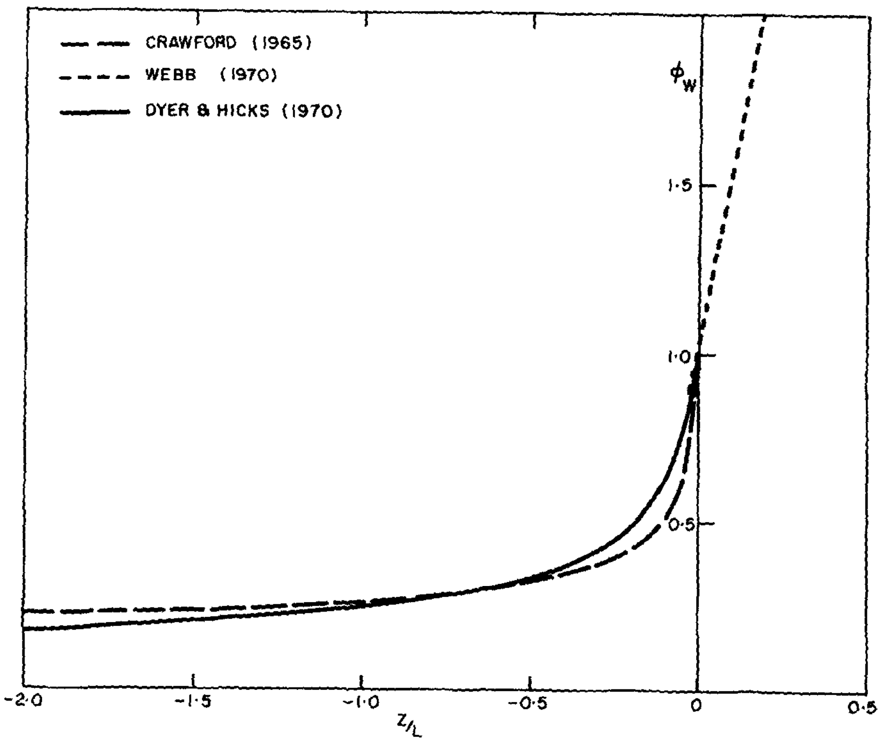

Fig. 3. ϕ_W plotted as a function of z/L. Crawford's (1965) data have been plotted assuming $\phi_M = (1 - 16(z/L))^{-1/4}$ and $k = 0.42$.

Swinbank's (1968) results come close to this in that only the u_* values were not determined directly but were obtained from a friction coefficient applied to a low-level wind. The difficulty here is that the variation of the friction coefficient with stability, which Swinbank felt could be neglected, depends on the form of the ϕ_M curve which finally emerges. A positive feed-back situation arises in which if the ϕ_M curve is presumed to depart fairly slowly from unity with increasing instability as predicted by the exponential wind profile, the stability variation of the friction coefficient is then also small, and a small variation of ϕ_M finally appears also. The reverse is true if a large variation is assumed. The only test of the final result is whether or not measurements taken at various heights mesh satisfactorily on a z/L plot. This is not a very sensitive test, particularly in view of the scatter which inevitably accompanies this kind of work.

Webb's (1970) analysis does not include any direct measurement of the eddy fluxes, and rests entirely on profile analyses. He assumes a log-linear form of profile, and sets out to determine the constant α (separated here into α_u and α_θ), and the stability range over which the log-linear form is valid. In order to determine α_u, he finds it necessary to take $\alpha_\theta = \alpha_u$, based on a later analysis apparently showing that K_H/K_M is constant over a wide range of Ri. This is not a serious weakness in the determination of α_u, and indeed Webb's case for the α_u values he finds is admirably argued. The technique calls for extremely accurate measurements of wind differences, and there is always the danger that results which do not reveal a satisfactory plot and are rejected on the grounds of a presumably instrumental error, *may* be due to the normal statistical variation of atmospheric behaviour.

Because of need for extreme precision, the determination of α_θ by this means in near-neutral conditions (i.e., $\Delta\theta \simeq 0$) is much more difficult, and indeed Webb presents only a few results for α_θ obtained in stable conditions.

In order to extend the stability range of the discussion, Webb examined the variation of K_H/K_M with stability by the technique of plotting $(\Delta u/\Delta\theta)/(\Delta u/\Delta\theta)_{0.03}$ against Ri where the subscript 0.03 refers to Ri. Unfortunately, this technique must be somewhat insensitive since a double ratio is now being taken. For example, Dyer and Hicks (1970) point out that their results for θ_M and ϕ_H, leading to quite a different variation of K_H/K_M with stability, fit Webb's graph equally well, indicating the inherent insensitivity of the method. Thus while the two versions of ϕ_M are virtually identical over the stability range involved, the difference in the ϕ_H forms are not revealed.

Turning now to a comparison of the Dyer and Hicks (1970) and the Businger *et al.* (1971) results, for the unstable situation, it is seen that the variation of the ϕ_M and ϕ_H forms with z/L is remarkably similar. The only difference of any real significance stems from the different values taken for the von Karman constant. Dyer and Hicks did not specifically determine this constant, and their results did not permit a traverse across neutral into the stable region. However, their basic technique was identical to that of Businger *et al.* (1971) and extended down to a $(-z/L)$ value of <0.01. As can be seen from Figure 1, the approach of ϕ_M to unity at $z/L = 0$ confirms their choice of $k = 0.41$.

Businger *et al.* (1971) in arriving at their final form of the data, reduced the wind shears by 10% because of alleged overspeeding, and reduced the drag-plate measurements of stress by 33% to provide harmony with the eddy-flux measurements of stress. Whilst the authors' reasons for applying these corrections are cogently argued, it is difficult to avoid speculating that if these corrections were not necessary, the von Karman constant would have been found to be 0.39, ϕ_H at neutral would have been 1.00, and hence $K_H = K_M$.

This aspect of the comparison of these most recent results remains the only significant area of controversy but because of the magnitude of the difference, a rather important one. Hopefully, it will be resolved by a definitive experiment before very long.

The result that $\phi_W = (1-16\,z/L)^{-1/2}$ for $z/L<0$ as expressed by Dyer and Hicks (1970) is unmatched by any other data presented. It is however entirely consistent with the shape-function analysis of Swinbank and Dyer (1967) (see also Dyer, 1967) which implies $\phi_W = \phi_H$. There is also considerable support offered by the elegant analysis of Crawford (1965) using the dimensionless E^* as a function of Ri. In Figure 3, Crawford's plot of E^* vs Ri has been transformed assuming that $\mathrm{Ri} = z/L$ (for $z/L<0$) and $\phi_M = (1-16(z/L))^{-1/4}$. There is a minor discrepancy with the log-linear form of Webb, in that Dyer and Hicks' result implies $\alpha_W = 8$, but in view of the limited range of the log-linear form, this is no serious matter.

On the stable side, the experimental material is rather limited, but apart from the Businger *et al.* (1971) result for ϕ_H which relates to some of the earlier discussion, there does not appear to be any major differences of opinion.

4. Concluding Remarks

Overall it could be argued that the most convincing flux-gradient description, and one which is reasonably consistent with the long-established body of literature on the subject is that provided by Dyer and Hicks (1970) for the unstable region, namely

$$\phi_H = \phi_W = (1 - 16(z/L))^{-1/2}, \qquad \phi_M = (1 - 16(z/L))^{-1/4},$$

and

$$\phi_H = \phi_W = \phi_M = 1 + 5(z/L)$$

for the stable region.

The carefully determined results of Businger *et al.* (1971) remain a difficulty which calls for considerable clarification.

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References

Busch, N. E.: 1973, 'The Surface Boundary Layer', *Boundary-Layer. Meteorol.* **4**, 213–240.

Businger, J. A., Wyngaard, J. C., Izumi, I., and Bradley, E. F.: 1971, 'Flux-Profile Relationships in the Atmospheric Surface Layer', *J. Atmospheric Sci.* **28**, 181–189.

Crawford, T. V.: 1965, 'Moisture Transfer in Free and Forced Convection', *Quart. J. Roy. Meteorol. Soc.* **91**, 18–27.

Dyer, A. J.: 1967, 'The Turbulent Transport of Heat, and Water Vapour in on Unstable Atmosphere', *Quart. J. Roy. Meteorol. Soc.* **93**, 501–508.

Dyer, A. J. and Hicks, B. B.: 1970, 'Flux-Gradient Relationships in the Constant Flux Layer', *Quart. J. Roy. Meteorol. Soc.* **96**, 715–721.

Lumley, J. L. and Panofsky, H.: 1964, *The Structure of Atmospheric Turbulence*, Interscience Publishers, New York.

Monin, A. S. and Obukhov, A. M.: 1954 'Basic Regularity in Turbulent Mixing in the Surface Layer of the Atmosphere', *Akad. Nauk. S.S.S.R. Trud. Geofiz. Inst.* **24**, 151.

Monin, A. S. and Yaglom, A. M.: 1971, *Statistical Fluid Mechanics, Mechanics of Turbulence*, MIT Press, Cambridge.

Swinbank, W. C.: 1964, 'The Exponential Wind-Profile', *Quart. J. Roy. Meteorol. Soc.* **90**, 119–135.

Swinbank, W. C.: 1968, 'A Comparison Between Predictions of Dimensional Analysis for the Constant Flux Layer and Observations in Unstable Conditions', *Quart. J. Roy. Meteorol. Soc.* **94**, 460–467.

Swinbank, W. G. and Dyer, A. J.: 1967, 'An Experimental Study in Micrometeorology', *Quart. J. Roy. Meteorol. Soc.* **93**, 494–500.

Webb, E. K.: 1970, 'Profile Relationships: the Log-Linear Range, and Extension to Strong Stability', *Quart. J. Roy. Meteorol. Soc.* **96**, 67–90.