Environmental Fluid Dynamics: Radiation Model

Dr. Jeremy A. Gibbs

Department of Mechanical Engineering
University of Utah

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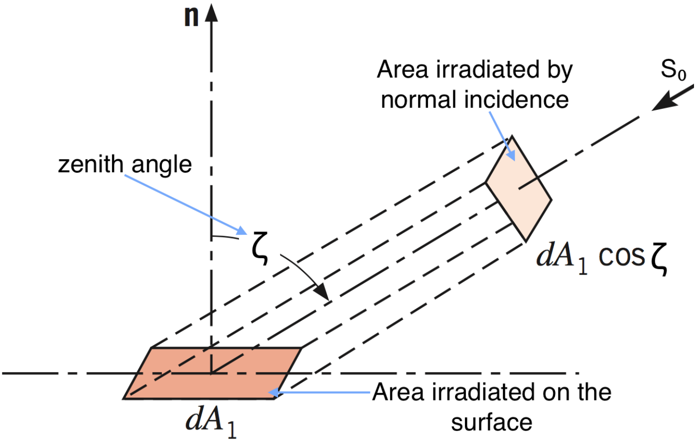
Overview

A Simple Model for the Computation of Radiation on a Slope

Radiation Model: Overview



- Flux density on a horizontal surface

$$S = S_0 \cos\zeta \rightarrow \text{Lambert's Cosine Law}$$

where $S_0 = 1367\ \mathrm{W\ m^{-2}}$ is the solar constant

Radiation Model: Overview

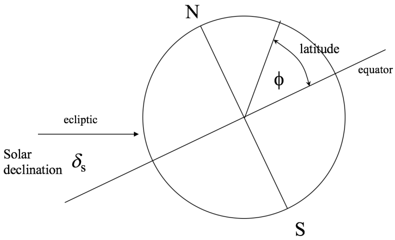
We want to consider a more general case of radiation on a slope of arbitrary:

- angle
- orientation
- time of day
- location on Earth
- time of year

Let's build a model:

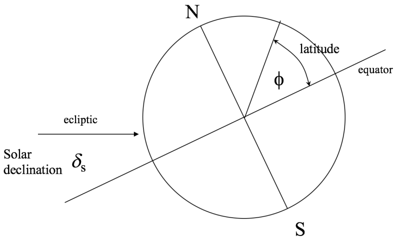
- Consider seasonal effects (Earth's orbit)
- Daily effects (local sun angle and azimuth)
- Slope (mountains, walls, etc)
- Atmospheric composition

Radiation Model: Seasonal Effects



- ϕ - latitude
- ϕ_r - tilt of the Earth's axis relative to the ecliptic (orbital plane of the Earth around the sun)
- $\phi_r = 23.45° = 0.409$ rad corresponds to the latitude of the tropics (Capricorn and Cancer)

Radiation Model: Seasonal Effects



Solar declination δ_s

- Angle between the ecliptic and the equator as the Earth rotates around the sun
- Summer and Winter

$$\underbrace{-23.45^\circ}_{\text{winter solstice}} \lesssim \delta_s \lesssim \underbrace{23.45^\circ}_{\text{summer solstice}}$$

- Spring and Autumn
  $\delta_s = 0$ means sun is directly overhead at equator: equinox
  vernal (Mar 19-21)
  autumnal (Sept 22-24)

Radiation Model: Seasonal Effects

Solar declination δ_s

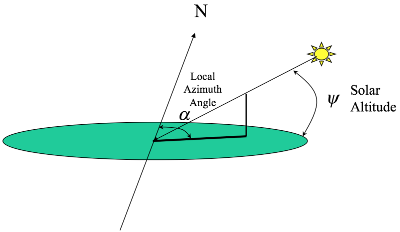
- For a circular orbit:

$$\delta_s = \phi_r \cos\left[\frac{C(d-d_r)}{d_y}\right]$$

where

- $C = 360°$ or $2\pi\ \mathrm{rad}$
- $d =$ day of the year (Julian date 1-365, or 366)
- $d_r =$ summer solstice (June 22, 173 JD, 20-22)
- $d_y =$ 365 or 366 days
- $\phi_r =$ tilt of Earth's axis relative to the ecliptic $(23.45°)$

Radiation Model: Daily Effects



- ψ - local sun elevation angle ("solar altitude")
- α - local azimuth angle (> 0 clockwise)
- ϕ - latitude (> 0 N hemisphere)
- λ_e - longitude (> 0 W of prime meridian)

Radiation Model: Daily Effects

Spherical relationships for local elevation angle

$$\sin\psi = \sin\phi\sin\delta_s - \cos\phi\cos\delta_s\cos\underbrace{\left[\frac{Ct_{\mathsf{UTC}}}{t_d} - \lambda_e\right]}_{B}$$

where

- $C = 360°$ or 2π rad
- t_{UTC} = local time + hours to UTC
- t_d = 24 hours
- λ_e = longitude (> 0 west from prime meridian)

Radiation Model: Daily Effects

Spherical relationships for local azimuth angle

$$\cos\alpha = \frac{\sin\delta_s - \sin\phi\cos\zeta}{\cos\phi\sin\zeta}$$

where

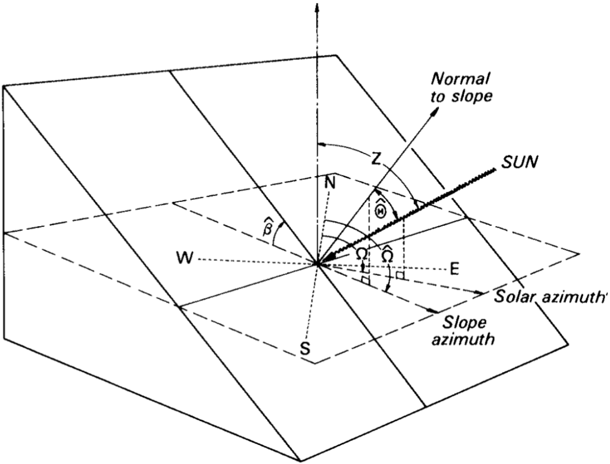
- the zenith angle ζ is given by

$$\begin{aligned}\zeta &= 90^\circ - \psi \\ &= \frac{\pi}{2} - \psi \text{ (radians)}\end{aligned}$$

- Correction in afternoon for sun setting in west is

$$\alpha' = C - \alpha$$

Radiation Model: Slope Effects



Z – *Zenith angle*
$\hat{\beta}$ – *Slope angle*
Ω – *Solar azimuth angle*
$\hat{\Omega}$ – *Slope azimuth angle*
$\hat{\Theta}$ – *Angle of incidence (between Sun and the normal to the slope)*

Radiation Model: Slope Effects

Spherical relationships for slope

$$\cos\hat{\theta} = \cos\hat{\beta}\cos\zeta + \sin\hat{\beta}\sin\zeta\cos(\alpha - \hat{\Omega})$$

Recall that for a flat surface

$$S = S_0 \cos\zeta$$

While for a sloping surface (absent scattering/absorption)

$$S = S_0 \cos\hat{\theta}$$

Radiation Model: Net Shortwave

Net shortwave radiation

$$R_s = R_{s\downarrow}(1 - a)$$

if $|R_L| \ll R_s$ (true under clear skies during the day), then

$$R_N = R_{s\downarrow}(1 - a)$$

In reality, we must also consider atmospheric transmissivity

$$\begin{aligned} R_{s\downarrow} &= S_0 T_r \sin\phi \\ &= S_0 T_r \cos\zeta \end{aligned}$$

where

- T_r = transmissivity (net sky transmissivity)
- S_0 = solar constant $\simeq 1367 \text{ W m}^{-2}$

Radiation Model: Net Shortwave

Transmissivity Depends on

- path length through atmosphere
- atmospheric absorption
- cloudiness

Simple model

$$T_r = (0.6 + 0.2\sin\psi)(1 - 0.4\sigma_H)(1 - 0.7\sigma_M)(1 - 0.4\sigma_L)$$

where σ_H, σ_M, and σ_L are cloud cover fractions for high-, mid-, and low-level clouds, respectively $(0 \le \sigma_i \le 1)$

Net longwave radiation

$$R_L = R_{L\downarrow} + R_{L\uparrow}$$

where

$$R_{L\uparrow} = -\epsilon\sigma T^4$$

However, it is harder to model $R_{L\downarrow}$, so we will model the net longwave radiation R_L

$$R_L = b(1 - 0.1\sigma_H - 0.3\sigma_M - 0.6\sigma_L)$$

where $b = -98.5 \text{ W m}^{-2}$

Net radiation model

$$
\begin{aligned}
R_N &= R_{S\downarrow} + R_{S\uparrow} + R_{L\downarrow} + R_{L\uparrow} \\
&= S_0 T r \cos\zeta(1-a) + R_L && \text{during day} \\
&= R_L (R_L < 0) && \text{during night}
\end{aligned}
$$

Radiation Model: Net Radiation Model

If asked to model R_N for a specific location, on a specific day, at a specific time, with specific cloud cover and surface properties, follow these steps

- Compute δ_s
- Find solar elevation angle ψ
- Solve for transmissivity T_r
- Compute R_S and R_L contributions
- Compute R_N