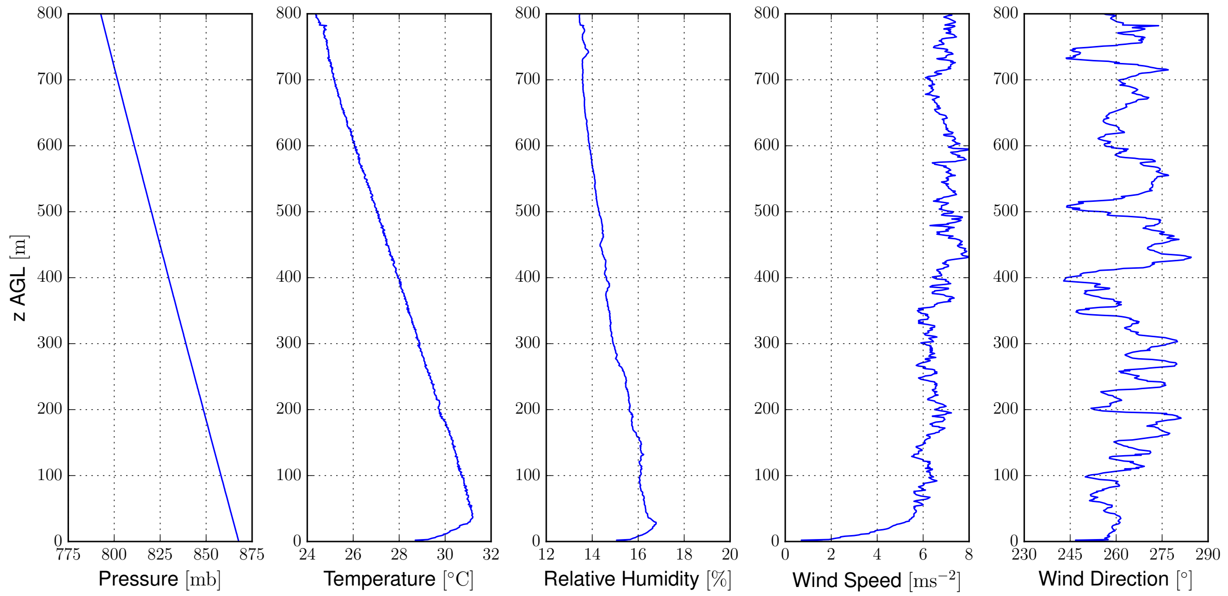
Environmental Fluid Dynamics

ME EN 7710 Homework #2 Solutions

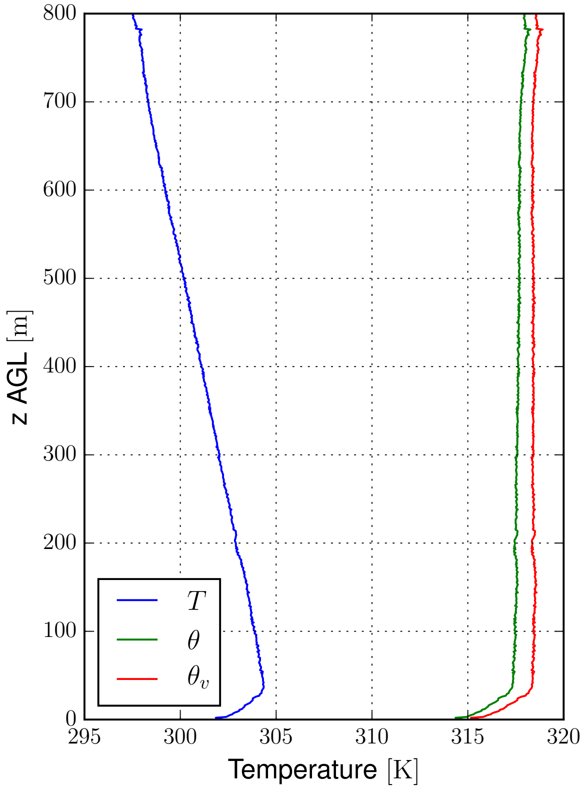
1.) Boundary Layer Profiles

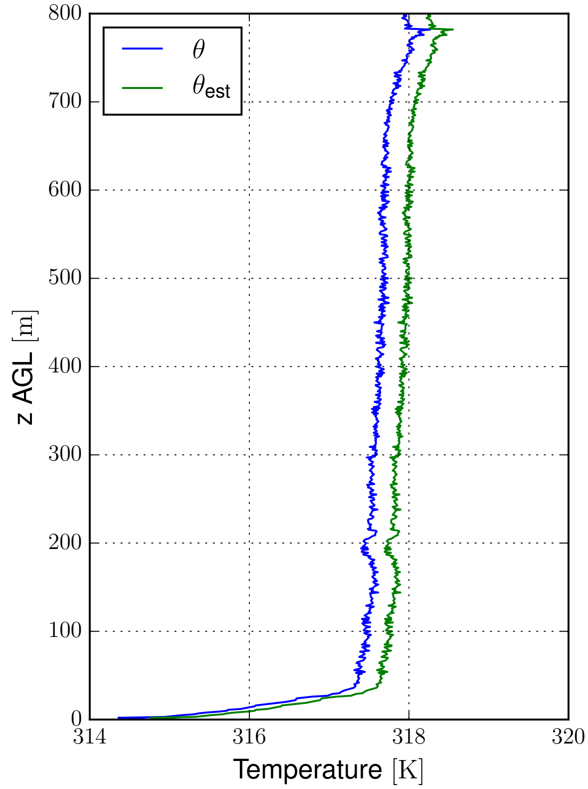
Given the provided ballon data (http://gibbs.science/efd/homework/balloon_data.txt), perform the following:

(a) Plot vertical profiles of all the raw variables on a single page with clearly labeled axes. Note any interesting features.



- Pressure decreases linearly with height
- Temperature increases in the lowest 50 m and then decreases linearly with height.
- Relative humidity also increases in the lowest 50 m and then decreases nearly linearly with height, although the slope is not as large as for pressure and temperature.
- There is a large wind speed gradient int he lowest 50 m, and then wind speed becomes quasi-constant with height.
- Wind speed seems to vary between $260° \pm 15°$ in the vertical.





(b) Calculate and plot the (i) temperature, (ii) potential temperature, and (iii) virtual potential temperature. Compare and discuss the potential temperature and virtual potential temperatures. Why do they differ?

- The temperature and potential differ because the potential temperature describes the temperature if it were brought adiabatically to some reference level ($p_0 = 1000$ mb). The lowest-level temperature from the balloon is at ~ 870 mb, so we would expect a parcel to warm adiabatically as it is brought to 1000 mb).
- The virtual potential temperature is the potential temperature that dry air would need to attain in order to have the same density as the moist air at the same pressure. Since moist air is less dense than dry air for the same T and p, θ_v is always greater than θ.

(c) Compare the potential temperature calculations using the balloon-based pressure measurements with $\theta(z) \approxeq T(z) + \Gamma z$. Calculate and plot the difference. What is the difference?

- The difference is rather small throughout the entire depth of the balloon measurements (on the order of 0.3 K).
- This indicates that the atmosphere is only roughly dry adiabatic (slightly smaller lapse rate).

2.) The Laminar Ekman Layer Above a Rigid Surface

The simplified momentum equations:

$$-fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}$$
$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2}$$

can be further simplified by expressing pressure gradients in terms of geostrophic velocity components as:

$$-f(v - V_g) = \nu\frac{\partial^2}{\partial z^2}(u - U_g) \tag{1}$$
$$f(u - U_g) = \nu\frac{\partial^2}{\partial z^2}(v - V_g) \tag{2}$$

(a) Assuming U_g and V_g are height independent, solve Eqs. (1) and (2) subject to the following boundary conditions:

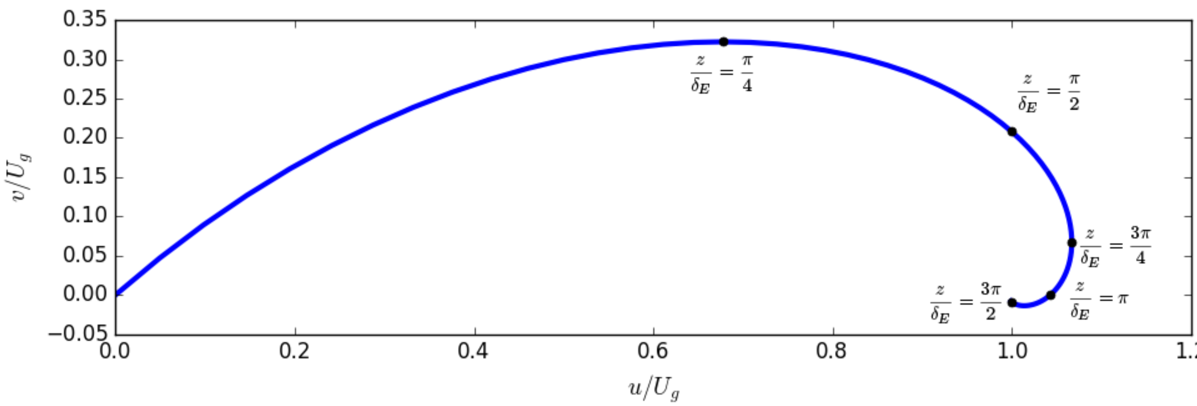
- $u(z{=}0) = v(z{=}0) = 0$
- $u(z \to \infty) = U_g$, $v(z \to \infty) = V_g$

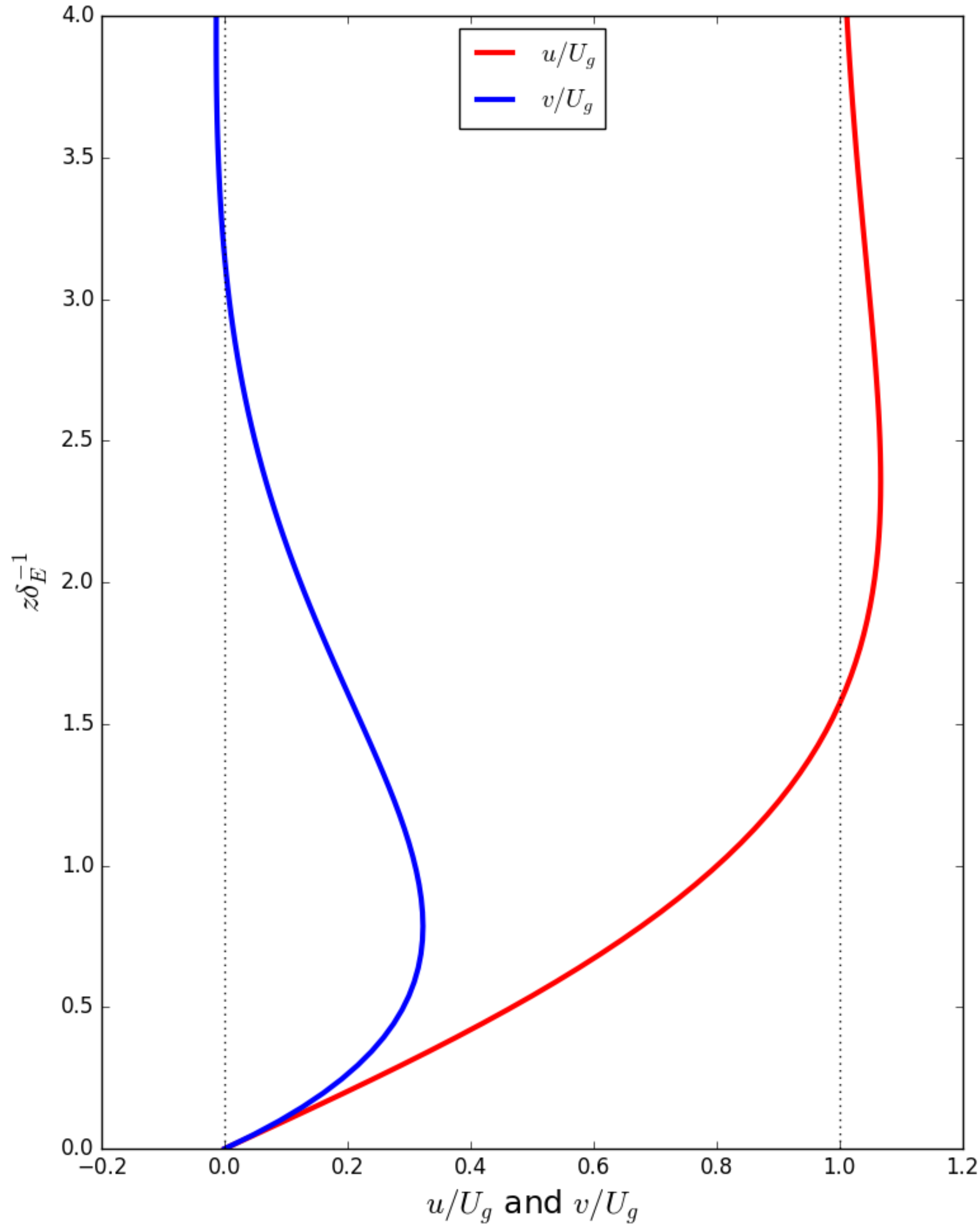
For the final solution, orient the x-axis with the geostrophic wind vector (i.e., $U_g = U$ and $V_g = 0$). The solutions should be in the form (where a is the inverse Ekman depth):

$$u = U[1 - e^{-az}\cos(az)] \qquad v = Ge^{-az}\sin(az)$$

- See Lecture 12 for the full derivation

(b) Plot your solution as a hodograph and as vertical profiles of u and v.





(c) Please explain the phenomena of Ekman pumping.

- Ekman pumping describes how friction induces a flow component toward low pressure at low levels, which leads to horizontal convergence into the low-pressure zone and thus rising motion.