ENVIRONMENTAL FLUID DYNAMICS

ME EN 7710 | Exam #1 | Take-Home Solutions

1.) [30 points] Turbulence Kinetic Energy

a.) [18 points] Start with the provided turbulent momentum flux equation and derive the balance equation for turbulence kinetic energy ($0.5\overline{u_i'u_i'}$). Be sure to assume incompressibility.

$$\frac{\partial(\overline{u_k'u_i'})}{\partial t} + \overline{u}_j\frac{\partial(\overline{u_k'u_i'})}{\partial x_j} = -\left[\overline{u_j'u_i'}\frac{\partial \overline{u}_k}{\partial x_j} + \overline{u_k'u_j'}\frac{\partial \overline{u}_i}{\partial x_j}\right] - \frac{\partial(\overline{u_k'u_j'u_i'})}{\partial x_j} + \overline{u_k'b'}\delta_{i3} + \overline{u_i'b'}\delta_{k3} - \left[\frac{\partial(\overline{u_k'\Pi'})}{\partial x_i} + \frac{\partial(\overline{u_i'\Pi'})}{\partial x_k} - \overline{\Pi'\left(\frac{\partial u_k'}{\partial x_i} + \frac{\partial u_i'}{\partial x_k}\right)}\right] + \nu\frac{\partial^2(\overline{u_k'u_i'})}{\partial x_j^2} - 2\nu\overline{\frac{\partial u_k'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

See Lecture 18, slides 4-7 for the derivation

$$\underbrace{\frac{\partial \overline{e}}{\partial t}}_{1} = -\underbrace{\overline{u}_j\frac{\partial \overline{e}}{\partial x_j}}_{2} - \underbrace{\overline{u_j'u_i'}\frac{\partial \overline{u}_i}{\partial x_j}}_{3} - \underbrace{\frac{\partial(\overline{u_j'e})}{\partial x_j}}_{4} + \underbrace{\overline{u_i'b'}\delta_{i3}}_{5} - \underbrace{\frac{\partial(\overline{u_i'\Pi'})}{\partial x_i}}_{6} + \underbrace{\nu\frac{\partial^2\overline{e}}{\partial x_j^2}}_{7} - \underbrace{\nu\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}}_{8}$$

b.) [6 points] Identify the meaning of each of the terms in your final equation.

(1) Storage of tke
(2) Advection of tke by the mean wind
(3) Production of tke by the mean wind shear
(4) Transport of tke by turbulent motions (turbulent diffusion)
(5) Production/destruction of tke by buoyancy
(6) Transport of tke by pressure (pressure diffusion)
(7) Molecular diffusion of tke
(8) Viscous dissipation of tke

c.) Simplify your turbulence kinetic energy equation for the following cases:

- [2 points] Horizontally homogeneous + no subsidence ($\overline{w} = 0$)

$$\frac{\partial \overline{e}}{\partial t} = -\overline{w'u'}\frac{\partial \overline{u}}{\partial z} - \overline{w'v'}\frac{\partial \overline{v}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} + \overline{w'b'} - \frac{\partial(\overline{w'\Pi'})}{\partial z} - \epsilon$$

- [2 points] Horizontally homogeneous + steady state

$$\overline{w}\frac{\partial \overline{e}}{\partial z} = -\overline{w'u'}\frac{\partial \overline{u}}{\partial z} - \overline{w'v'}\frac{\partial \overline{v}}{\partial z} - \overline{w'w'}\frac{\partial \overline{w}}{\partial z} - \frac{\partial (\overline{w'e})}{\partial z} + \overline{w'b'} - \frac{\partial (\overline{w'\Pi'})}{\partial z} - \epsilon$$

- [2 points] Horizontally homogeneous + steady state + no subsidence + neutral stability

$$0 = -\overline{w'u'}\frac{\partial \overline{u}}{\partial z} - \overline{w'v'}\frac{\partial \overline{v}}{\partial z} - \frac{\partial (\overline{w'e})}{\partial z} - \frac{\partial (\overline{w'\Pi'})}{\partial z} - \epsilon$$

2.) [20 points] Radiation

a.) [10 points] Calculate the total radiative flux for a black body surface at a temperature of 302 K, emitting radiation over the following wavelengths: $5\ \mu\text{m}$ to $40\ \mu\text{m}$. Feel free to integrate numerically. What type of radiation is this considered?

$$R = \int_{\lambda_1}^{\lambda_2} R_\lambda d\lambda$$

$$R = \frac{2\pi hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{b\lambda T}\right) - 1\right] - 1$$
$$h = 6.626 \times 10^{-34}$$
$$c = 2.9979 \times 10^{8}$$
$$b = 1.381 \times 10^{-23}$$

$$\boxed{R = 439.6\ \text{W m}^{-2}}$$

This is in the longwave portion of the spectrum.

b.) [10 points] Calculate the wavelength of maximum radiant energy.

$$\lambda = \frac{2898}{T}$$
$$\lambda = \frac{2898}{302}$$
$$\boxed{\lambda = 9.6\ \mu\text{m}}$$

3.) [20 points] Heat Budget

In the afternoon, the incoming short wave radiation was measured at $50\ \mathrm{m}$ above ground to be $750\ \mathrm{W\ m^{-2}}$. The layer cools at a rate of $dT/dt = 0.02°\mathrm{C/day}$. Calculate the latent and sensible heat flux for the following environments (assume that the net longwave radiation is very small and that the flux into the submedium is negligible):

a.) [10 points] a desert surface (albedo = 0.3, Bowen Ratio = 10)

$$R_N = H_S + H_L + \cancel{H_G} + \Delta H_S$$

where

$$R_N = (1-\alpha)R_{s\downarrow}$$

and

$$\Delta H_S = \int \rho C T dz \approx 0.0116\ \mathrm{W\ m^{-2}}$$

$$(1-\alpha)R_{s\downarrow} = H_S + H_L + \Delta H_S$$
$$(1-\alpha)R_{s\downarrow} = H_S + \frac{H_S}{\mathrm{BR}} + \Delta H_S$$
$$(1-\alpha)R_{s\downarrow} = H_S\left(1 + \frac{1}{\mathrm{BR}}\right) + \Delta H_S$$
$$H_S = \frac{(1-\alpha)R_{s\downarrow} - \Delta H_S}{\left(1 + \frac{1}{\mathrm{BR}}\right)}$$
$$\boxed{H_S = 477.26\ \mathrm{W\ m^{-2}}}$$
$$\boxed{H_L = H_S/\mathrm{BR} = 47.72\ \mathrm{W\ m^{-2}}}$$

b.) [10 points] an irrigated crop surface (albedo=0.18, Bowen Ratio = 0.2)

Substituting the new values into the expressions above, yields:

$$\boxed{H_S = 102.49\ \mathrm{W\ m^{-2}}}$$
$$\boxed{H_L = H_S/\mathrm{BR} = 512.49\ \mathrm{W\ m^{-2}}}$$

4.) [30 points] Sensible, Latent, and Buoyancy Fluxes

a.) [20 points] The turbulent buoyancy flux $\overline{w'b'}$ can be defined as $\frac{g}{\theta_v}\overline{w'\theta_v'}$. Consider just the $\overline{w'\theta_v'}$ portion of the equation. Using the definition of virtual temperature, Reynolds averaging, appropriate assumptions, and the relationships derived in class, derive an expression for $\overline{w'\theta_v'}$ in terms of the kinematic sensible heat flux ($\overline{w'T'}$) and the kinematic latent heat flux ($\overline{w'q'}$). Please show all work and clearly identify assumptions.

Many people wrote out long derivations (that's fine), but here is the most compact version used in atmospheric sciences:

$$\theta_v = \theta + 0.61\theta q$$

A common assumption for the BL is to linearize the equation as

$$\theta_v = \theta + 0.61\theta_0 q$$

where θ_0 is a constant reference value. Expanding yields:

$$\overline{\theta_v} + \theta_v' = \overline{\theta} + \theta' + 0.61\theta_0\overline{q} + 0.61\theta_0 q'$$

If we take into account that

$$\overline{\theta_v} = \overline{\theta + 0.61\theta_0 q} = \overline{\theta} + 0.61\theta_0\overline{q}$$

then we can write that

$$\theta_v' = \theta' + 0.61\theta_0 q'$$

Now we showed in class on slide 40 of Lecture 11 that:

$$\frac{\rho'}{\overline{\rho}} \approx -\frac{T'}{\overline{T}} \approx -\frac{\theta'}{\overline{\theta}}$$

That means that we generally assume that $\theta' \approx T'$ and the reference values do not greatly differ. So,

$$\theta_v' = \theta' + 0.61\theta_0 q' \approx T' + 0.61T_0 q'$$

Thus, finally:

$$\boxed{\overline{w'\theta_v'} \approxeq \overline{w'T'} + 0.61T_0\overline{w'q'}}$$

b.) [10 points] Using your relationship from part (a), please explain the effect of moisture on the buoyancy flux.

Given that expression, we can see that moisture acts to enhance the buoyancy flux (i.e., inclusion of moisture is important to properly account for the buoyancy flux).