Environmental Fluid Dynamics

ME EN 7710 | Exam #1 | Take-Home Portion

1.) [30 points] Turbulence Kinetic Energy

a.) [18 points] Start with the provided turbulent momentum flux equation and derive the balance equation for turbulence kinetic energy ($0.5\overline{u_i'u_i'}$). Be sure to assume incompressibility.

$$\frac{\partial(\overline{u_k'u_i'})}{\partial t} + \overline{u}_j \frac{\partial(\overline{u_k'u_i'})}{\partial x_j} = -\left[\overline{u_j'u_i'}\frac{\partial \overline{u}_k}{\partial x_j} + \overline{u_k'u_j'}\frac{\partial \overline{u}_i}{\partial x_j}\right] - \frac{\partial(\overline{u_k'u_j'u_i'})}{\partial x_j}$$
$$+ \overline{u_k'b'}\delta_{i3} + \overline{u_i'b'}\delta_{k3}$$
$$- \left[\frac{\partial(\overline{u_k'\Pi'})}{\partial x_i} + \frac{\partial(\overline{u_i'\Pi'})}{\partial x_k} - \overline{\Pi'\left(\frac{\partial u_k'}{\partial x_i} + \frac{\partial u_i'}{\partial x_k}\right)}\right] + \nu\frac{\partial^2(\overline{u_k'u_i'})}{\partial x_j^2} - 2\nu\overline{\frac{\partial u_k'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

b.) [6 points] Identify the meaning of each of the terms in your final equation.

c.) Simplify your turbulence kinetic energy equation for the following cases:

- [2 points] Horizontally homogeneous + no subsidence ($\overline{w} = 0$)
- [2 points] Horizontally homogeneous + steady state
- [2 points] Horizontally homogeneous + steady state + no subsidence + neutral stability

2.) [20 points] Radiation

a.) [10 points] Calculate the total radiative flux for a black body surface at a temperature of 302 K, emitting radiation over the following wavelengths: 5 μm to 40 μm. Feel free to integrate numerically. What type of radiation is this considered?

b.) [10 points] Calculate the wavelength of maximum radiant energy.

3.) [20 points] Heat Budget

In the afternoon, the incoming short wave radiation was measured at 50 m above ground to be $750\ \mathrm{W\ m^{-2}}$. The layer cools at a rate of $dT/dt = 0.02°\mathrm{C}$/day. Calculate the latent and sensible heat flux for the following environments (assume that the net longwave radiation is very small and that the flux into the submedium is negligible):

a.) [10 points] a desert surface (albedo = 0.3, Bowen Ratio = 10)

b.) [10 points] an irrigated crop surface (albedo=0.18, Bowen Ratio = 0.2)

4.) [30 points] Sensible, Latent, and Buoyancy Fluxes

a.) [20 points] The turbulent buoyancy flux $\overline{w'b'}$ can be defined as $\frac{g}{\theta_v}\overline{w'\theta_v'}$. Consider just the $\overline{w'\theta_v'}$ portion of the equation. Using the definition of virtual temperature, Reynolds averaging, appropriate assumptions, and the relationships derived in class, derive an expression for $\overline{w'\theta_v'}$ in terms of the kinematic sensible heat flux ($\overline{w'T'}$) and the kinematic latent heat flux ($\overline{w'q'}$). Please show all work and clearly identify assumptions.

b.) [10 points] Using your relationship from part (a), please explain the effect of moisture on the buoyancy flux.